Frequency Stabilization
of a 732 nm Diode Laser

Mathieu Alloing

Master 1/Magistère 2 de Physique Fondamentale,
Université Paris XI, France

Supervisor : Juergen Eschner

At : ICFO The Institute of Photonic Sciences
Mediterranean Technology Park
Av. del Canal Olmpic s/n
08860 Castelldefels (Barcelona), Spain

University supervisor : Corinne Augier

5 May - 8 August 2008
Abstract:

Towards the realization of a quantum computer, single atoms/ions appear highly interesting as they serve as ideal quantum memory namely static quantum bits (qubits). Quantum state manipulations used for quantum processing need high stabilized laser frequencies to interact with the quantum states of trapped single atoms/ions. This report then deals with the frequency stabilization of a 732 nm laser onto a Fabry-Pérot cavity for spectroscopy and quantum state manipulations of a trapped $^{40}\text{Ca}^+$ ion. The Pound-Drever-Hall method used for the stabilization will be explained as well as the key role of Fabry-Pérot cavities. The necessary electronic and optical setups I implemented during my stay are also presented. The results of my work, namely the linewidth, frequency stability and output power, are measured and compared to reference results to conclude on the efficiency of the setup.

Résumé:

Dans l’objectif de réaliser un ordinateur quantique, ions ou atomes isolés se sont révélés très intéressants comme mémoire quantique c’est-à-dire bits quantiques statiques (ou qubits). Les manipulations d’états quantiques utilisées pour la computation quantique requièrent des fréquences laser hautement stabilisées pour intégrer avec les niveaux quantiques de l’ion ou de l’atome. Ce rapport traite donc de la stabilisation en fréquence d’un laser de 732 nm sur une cavité Fabry-Pérot pour la spectroscopie et la manipulation des états quantiques d’un ion $^{40}\text{Ca}^+$ piégé. La méthode connue sous le nom de Pound-Drever-Hall employée pour la stabilisation, sera expliquée ainsi que le rôle clef des cavités Fabry-Pérot. Les montages électroniques et optiques que j’ai installé au cours de mon stage seront également présentés. Les résultats de mon travail, à savoir la puissance envoyée sur l’ion, la largeur de raie et la stabilité en fréquence du laser, sont comparées avec des résultats de référence pour conclure sur l’efficacité du montage.

ICFO and the “Quantum information and quantum optics with single trapped atoms” group:

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The “Quantum information and quantum optics with single trapped atoms” group where I effectuated my stay is led by Juergen Eschner and composed by 11 permanent physicists. Research fields are to study experimentally the quantum interaction of trapped, laser-cooled ions and atoms with laser light, aiming at its application for processing and transmitting quantum information. Particular emphasis is on the interaction between single trapped ions over long distances through optical fields, and on techniques for storing single neutral atoms.
0. GENERAL INTRODUCTION

Nowadays many groups world-wide are working in the field of quantum information processing with the goal of building a “quantum computer”. In a quantum computer, information is stored in a register consisting of a number of qubits (noted $|0\rangle$ and $|1\rangle$ by analogy with 0 and 1), which are two-state systems such as two-level atoms or spin-particles. The power of quantum computation is due to the fact that in contrast to a classical computer the register of a quantum computer can be prepared in a superposition of states which are then processed in parallel. A number of algorithms have been discovered that allow for the solution of certain problems more efficiently by a quantum computer than by any classical computer. The factorization of large numbers, and the search of unsorted data-bases are probably the most well known examples.

For this purpose Quantum Optics became increasingly important since light-matter interfaces are the most important building blocks of a quantum network, a network where quantum states between distant sites should be transferred. In this context, single atoms/ions serve as ideal quantum memory, as static quantum bits (qubits), whereas photons are ideal carriers of quantum information, so-called flying qubits.

In this field, a part of the group where I accomplished my stay plans to transfer photonic polarization states onto internal Zeeman substates of ions and then to realize a flying/storage qubit interface. For these experiments, two $^{40}\text{Ca}^+$ ions are trapped in two different radio-frequency linear traps (Paul’s trap) and frequency stabilized lasers are used for coherent manipulations of the $^{40}\text{Ca}^+$ states. Figure 0.1 presents the electronic ion levels and the laser wavelengths required for such manipulations.

![Figure 0.1: Electronic $^{40}\text{Ca}^+$ levels. Notice that all states split up into Zeeman substates.](image)

Seven lasers tuned at the seven possible wavelengths ensure an efficient control of all $^{40}\text{Ca}^+$ transitions and a large panel of experiments. More precisely, the 393 and 397 nm transitions are used to cool the trapped ions (Doppler cooling), to populate the $^{3}\text{D}_{3/2}$ and $^{3}\text{D}_{5/2}$ levels (with natural decay) and to visualize the ions (during the trapping or to see quantum jumps for instance). The 866, 854 and 850 nm transitions serve to control the population in the $^{3}\text{D}_{3/2}$ and $^{3}\text{D}_{5/2}$ levels and for qubit measurement.
Indeed, in the case of the $^{40}\text{Ca}^+$, the two Zeeman substates $m = \{-1/2, 1/2\}$ of the $^4\!{}^2\!^2\!^2\!S_{1/2}$ level can be associated with the logic states of a qubit. This can be assumed since as $^4\!{}^2\!^2\!^2\!S_{1/2}$ is a ground state the two substates have a sufficient lifetime to ensure the coherence of the qubit. The Zeeman substates $m = -1/2$ is selected as $|0\rangle$, the $m = 1/2$ with logic $|1\rangle$ and the transition between the two substates can be driven by a radiofrequency beam at about 10 MHz.

To readout quantum information one needs to measure populations of the $^4\!{}^2\!^2\!^2\!S_{\pm 1/2}$ states. In that purpose, a 732 nm laser can be used to drive the $^4\!{}^2\!^2\!^2\!S_{1/2}-^3\!^2\!^2\!^2\!D_{3/2}$ transition and to perform state detection. The principle of this read-out (shown in figure 0.2) is the following : assuming that the $m = -1/2$ Zeeman substate of $^4\!{}^2\!^2\!^2\!S_{1/2}$ is populated, a right circular polarized light at 732 nm illuminates to the ion. Following the transition selection rules the ion is driven to the $m = +1/2$ substate of $^3\!^2\!^2\!^2\!D_{3/2}$, a light linear polarized parallel to the quantization axis (defined by the magnetic field which causes the Zeeman splitting) at 866 nm is sent to the ion and drives it to the $m = +1/2$ substate of $^4\!^2\!^2\!^2\!P_{1/2}$. The ion will then decay to the ground state by spontaneous emission and emit a photon at 397 nm which can be measured and acts like a proof that the quantum memory was in the $|0\rangle$ state.

![Diagram of state detection principle](image)

**Fig. 0.2:** State detection principle. The $^{40}\text{Ca}^+$ levels are split up by Zeeman effect.

As the $^4\!{}^2\!^2\!^2\!S_{1/2}-^3\!^2\!^2\!^2\!D_{3/2}$ transition is an electric quadrupole transition three characteristics of the laser have to be improved to maximize the coupling between the ion and the light and thus to optimize the state detection (and the other manipulations using the 732 nm transition : cooling to the ground state, state preparation, efficient detection and state manipulation for instance) :

- high laser intensity,
- very stable frequency during a long period (typically some hours),
- narrow linewidth.

The purpose of my internship was to implement such improvements. The report of my work is organized as follows. First a presentation of the laser functioning and the necessary settings. Then some reminders about Fabry-Pérot cavities and the description the Pound-Drever-Hall method used for the frequency locking. In chapter three the setup for the laser frequency stabilization is explained and the linewidth of the laser is determined. Chapter five describes the setup used for the fine frequency adjustment. A comparison between the results and the values presented in the article [13] concludes this report.
1. OVERVIEW OF THE LASER BOXES AND SETTINGS

The laser used in this work is a TA 730-0101 from Toptica i.e. a DL100 with a taped amplifier.

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**Fig. 1.1:** Laser box.
The laser beam is emitted by a laser diode whose characteristics are: current threshold of 74.6 mA, operating current of 99.2 mA, center wavelength of 736.0 nm, a linewidth of a few MHz and a light output power of 20 mW at 25°C. The frequency output can be tuned by an extra resonant cavity formed by the back mirror of the laser diode and a diffraction grating as shown in the figure 1.1. The diffraction grating is set such that the -1 order goes back to the laser diode and the 0 order goes on the mirror in front of it to form the output beam.

This extra cavity acts like a filter for the frequencies originally produced by the laser diode (exactly like the action of a Fabry-Pérot described in the section 2.1.1). The frequencies which match the cavity length are amplified by the laser diode by stimulated absorption/emission what is the basic of laser functioning and what improves the diode efficiency. This set-up is called optical feedback.

The light reflected by the grating has to be perfectly aligned with the internal resonant cavity of the laser diode, what can be improved by setting the screws 2, 3 and 4. The optimization of the optical feedback is done by decreasing the laser current-threshold while displacing the spatial orientation of the grating. For that purpose, one can observe the laser beam (speckle aspect) while adjusting the screws. This leads to decrease the threshold of the diode while compensating the loss of current with the optical feedback. Usually the current threshold of the diode (namely the threshold where the diode starts lasing) can be decreased to less than 70 mV (often 68 mV) which insures an optimal functioning of the diode at operating current.

The length of the extra cavity can be modified with a piezo fixed on the grating. In the same manner as for a conventional Fabry-Pérot cavity it allows to scan the different frequency modes of the diode. The piezo is controlled by the Scan control module shown on the figure 3.1, the tuneable parameters are again the offset and the amplitude of the scan but a frequency of the piezo oscillation of about 2-3 kHz. An extra coarse offset is allowed by the screw 1, this is very useful since as the center wavelength of the diode is 736.0 nm the system has to be pushing close to its limits to obtain 732 nm. In fact the previous idea was to set the laser for the 729 nm transition (see the $^{40}\text{Ca}^+$ levels on the figure 0.1) but even with the system on the far limits of functioning only a wavelength of about 730 nm could be reached.

The laser frequency, i.e. its colour, strongly depends on the offset of the piezo, the current and the temperature of the laser diode. These three parameters are set to obtain optimum operations at the desired frequency of 732.389 nm.

An other important available setting is the trimpot called feed forward on the figure 3.1, the feed forward is an extra regulation which establishes the ratio between the piezo scan and current applied on the diode. This allows to increase the numbers of frequencies available in one mode.

After the grating a semi-reflecting mirror partly sends (about 8 mW) the laser beam on an back output of the laser box for the locking set-up which will be described in the section 3.1. Most of the power is reflected and then amplified by a taped amplifier which is a semiconductor diode using stimulated absorption/emission to amplify the incoming beam. The taped amplifier is supplied with a current of about 1 A that is more than 10 times the current applied onto the laser diode. Two lenses before and after the taped amplifier allows to focus the beam on the semiconductor and also to collimate the output beam.

Finally two Faraday isolators are used to avoid back reflected light, indeed back reflections could destabilize the laser diode and even destroy it.

At the end the output power sent on the AOM part (and then to the ion, see figure 4.2) is about 500 mW what is very powerful.
2. FREQUENCY STABILIZATION PRINCIPLE : POUND-DREVER-HALL METHOD

This chapter introduces to a powerful technique of laser frequency stabilization: the Pound-Drever-Hall method, which basics will be explained in the section 2.2. But first some reminders about one of the key part of the set-up used by this method: a Fabry-Pérot cavity.

2.1 Fabry-Pérot cavities

2.1.1 Gaussian beams and Fabry-Pérot cavities

To deal with laser beams, one needs a model of the emitted light which is given by the solutions of the paraxial form of the Helmholtz equation [3], [1]. In cartesian coordinates these solutions are the Hermite-Gauss modes often called Transverse Electric and Magnetic $(n, m)$ modes or TEM$_{n,m}$ modes. Lasers are usually operated with the lowest TEM$_{0,0}$ mode which is called fundamental transverse mode or gaussian mode. In this mode and for a circular symmetric beam the electric field can be written:

$$E(r, z) = E_0 \cdot \frac{\omega_0}{\omega(z)} \cdot e^{\frac{-r^2}{\omega(z)^2}} e^{-ikz - ik\frac{r^2}{2R(z)}} e^{i\zeta(z)}, \quad r = \sqrt{x^2 + y^2}$$

(2.1)

where:

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad z_R = \frac{\pi \omega_0^2}{\lambda}, \quad \zeta(z) = \arctan \left(\frac{z}{z_R}\right), \quad R(z) = z \left[1 + \left(\frac{z_R}{z}\right)\right]$$

$\omega(z)$ is the beam size (also called spot size, its shape is shown in figure 2.1) which is the radius to the z-axis at which the field amplitude drops to $1/e$ of its axis value. One can see that the spot size admits a minimum $\omega_0$ known as beam waist, $z_R$ is called the Rayleigh range and characterizes the points where $w(\pm z_R) = \sqrt{2}\omega_0$. $\zeta(z)$ is the Gouy phase or the longitudinal phase delay and $R(z)$ is the radius of curvature of the wavefront.

A standing gaussian beam confined between two spherical mirrors has to satisfy some boundary conditions imposed by the cavity (see 2.1), essentially conditions on the curvature of the wavefront matching on each mirror and global condition on the phase to create constructive interferences [11, 1].

![Fig. 2.1: Gaussian beam in a symmetric confocal cavity. $R_i$, $i = 1, 2$ are the mirrors radii of curvature.](image-url)

Useful variables for characterizing cavities are the resonator $g$ parameter, $g_i = 1 - \frac{L}{R_i}$, where $R_i$, $i = 1, 2$ are the mirrors radii of curvature and $L = z_2 - z_1$. 
The cavity used in this work is a symmetric confocal cavity with 2 identical mirrors satisfying $R_1 = R_2 = L$. Then, beam parameters are: $\omega_0^2 = \frac{L \lambda}{2 \pi}$ and $L = z_2 - z_1 = 2z_R$.

We have seen that the phase of a gaussian beam is $\Phi(z) = -kz - k \frac{\omega^2}{2R(z)} + \zeta(z))$, this phase can be extended to an $nm^{th}$ order Hermite-Gaussian mode and then the total phase shift from one end of the cavity to the other is $\Phi(z_2) - \Phi(z_1) = kL - (n + m + 1)(\zeta(z_2) - \zeta(z_1))$. It can be shown that $\zeta(z_2) - \zeta(z_1) = \arccos(\pm \sqrt{g_1 g_2})$ where + (resp. -) corresponds to $g_1, g_2 > 0$ (resp. $g_1, g_2 < 0$). Then to construct create interferences the global phase shift must satisfy:

$$kL - (n + m + 1) \arccos(\pm \sqrt{g_1 g_2}) = q\pi , \ (q, n, m) \in \mathbb{N}^3 , \ k = \frac{\omega}{c}$$

(2.2)

Finally for a near confocal cavity the resonance frequencies of the axial-plus-transverse modes are:

$$\omega_{q,n,m} = 2\pi (q + \frac{n + m + 1}{2}) \frac{c}{2L} , \ (q, n, m) \in \mathbb{N}^3$$

(2.3)

One can see that for instance 01 and 10 transverse modes associated with the $q$-th axial mode move out to fall exactly halfway between the $q$ and $q+1$ axial modes. Indeed for a confocal cavity and for $q \gg 1$ even-symmetric transverse modes $(00 , 02, 20, 11,...)$ $n, m$-th of a $q$-axial mode are exactly degenerate at the $n', m'$-th of the $q + 1$ axial mode frequencies and the odd-symmetric modes $(01 , 10, 21 , 12,...)$ are exactly degenerate at the half-axial positions midway between the axial mode locations. This induces that in a confocal cavity two consecutive interference fringes are separated by an effective frequency of $\frac{c}{4L}$; this number is called Free Spectral Range (FSR).

The size of the mirrors in a Fabry-Perot cavity is typically about 1 cm. As the spatial extension of high TEM modes rapidly increases out of this range only the lowest order can be coupled in effectively used cavities.

One can see by looking at the equation (2.3) that with a fixed laser frequencies falling in a cavity one can scan around these frequencies by varying the length of the cavity. This scanning is the basic method to visualize standing waves in a cavity. The output beams of such cavity will be described in the next section.

### 2.1.2 Parameters of a Fabry-Perot cavity

As shown in the Figure 2.2, the transmitted and reflected field of a cavity are labeled [9]:

$$E_t = tE_i$$

$$E_r = rE_i$$

with $E_i = E_0 e^{i\omega t}$ being the incident beam and $r$ and $t$ are respectively the reflection and transmission coefficients, where for mirrors without losses (mirror absorption neglected):

$$r^2 + t^2 = 1 , \ 0 \leq r, t \leq 1.$$  

Light traveling in the cavity will experienced a phase-shift $\phi$ of $\frac{2\pi}{L}$ per trip from a mirror to the other. So a factor $e^{i\phi}$ has to be added to the beam on each single trip. Additionally there is a phase shift of $\pi/2$ between the reflected and transmitted beams at a mirror (see [4]), this is taken in account by added a $\pi$-phase shift in the beams in the cavity (the calculation principle is presented in figure 2.2).

Then the total reflected beam $E_r$ for lossless mirrors is given by:

$$E_r = rE_0 + e^{i\pi}(te^{i\phi}r e^{i\phi}tE_0 + te^{i\phi}r e^{i\phi}r e^{i\phi}tE_0 + ...) = r\frac{e^{2i\phi} - 1}{1 - r^2 e^{2i\phi}} E_0$$

From this expression one can obtain the reflection transfer function of the cavity:

$$F_r(\omega) = \frac{E_r}{E_0} = r\frac{e^{2i\phi} - 1}{1 - r^2 e^{2i\phi}}$$

(2.4)

This term will be needed to derive the Pound-Drever-Hall error signal in the next section.
Then the reflected and transmitted intensity for a symmetric lossless cavity are:

\[
I_r = |E_r(\omega)|^2 = I_0 \frac{4r^2}{(1-r^2)^2} \frac{\sin^2(\phi)}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2(\phi)}
\] (2.5)

\[
I_t = |E_t(\omega)|^2 = I_0 \frac{4r^2}{(1-r^2)^2} \frac{1}{1 + \frac{4r^2}{(1-r^2)^2} \sin^2(\phi)}
\] (2.6)

where \(\phi = \frac{\omega L}{c} = \frac{\omega}{FSR}\). One can see from equations (2.5) and (2.6) that intensity variations with the frequency \(\omega\), or with cavity length \(L\) will produce identical outcomes which justifies the scan principle explained in the previous section. The figure 2.3 shows these two intensities.

The previous expressions allow one to introduces some very important cavity parameters. First the Free Spectral Range (FSR) clearly appears as the frequency distance between two consecutive maxima of the transmitted intensity (or minima for the reflected intensity) or between two consecutive TEM\(_{00}\) modes. Here \(FSR = \frac{c}{L}\) but we have previously seen that the effective FSR in a confocal cavity is half this value that is \(\frac{c}{2L}\).

The Finesse \(\tilde{F}\) is a measure for the average photon storage time in the cavity:

\[
\tilde{F} = 2\pi \tau FSR
\] (2.7)

where \(\tau\) is the lifetime of a photon in the cavity. It follows from this definition that the Finesse is directly related to the reflectivity \(r\) of the mirror, what can be seen by calculating the passive linewidth (passive refers to unlocked cavity, opposed to active linewidth when the cavity length is regulated) of the cavity from the reflected intensity:

\[
\tilde{\gamma} = \frac{\pi r}{1 - r^2}
\] (2.8)

The same calculation gives the relation between the linewidth \(\Delta\nu\) (also called Full Width at Half-Maximum) of the cavity and the FSR and the Finesse:

\[
\Delta\nu \cdot \tilde{F} = FSR
\] (2.9)

These parameters will be used for the laser linewidth measurement.
2. Frequency stabilization principle: Pound-Drever-Hall method

This section introduces the principle of the Pound-Drever-Hall (PDH) technique for stabilizing laser frequency. This method was invented in the group of John Hall [12]. Here only the basic analysis is explained, more information can be found in the articles [8, 5].

The idea behind the PDH method is the following: a laser’s frequency is measured with a Fabry-Perot cavity, and this measurement is fed back to the laser to suppress frequency fluctuations.

Most of the lasers used nowadays are tuneable, that is they present some input port into which one can feed an electrical signal and adjust the output frequency for instance by modifying the resonant cavity length of the laser.

As seen on the previous section the Fabry-Pérot cavity acts as a filter, with transmission lines, or resonances, spaced evenly in frequency every Free Spectral Range, so the idea is to set the laser frequency on a cavity resonance, measure the reflected intensity and hold it at zero. The problem is that the intensity of the reflected beam is symmetric about resonance. If the laser drifts out of the resonance with the cavity, it’s impossible to say just by looking at the reflected intensity whether the laser frequency needs to be increased or decreased to bring it back onto resonance. However the derivative of the reflected intensity is antisymmetric about resonance. With a measurement of this derivative it can be possible to get an error signal which can be used to lock the laser.

This can be achieved by modulating the laser frequency and by looking at the response of the reflected beam. The basic setup needed for the PDH method is shown in the figure 2.4.

Here the phase of the incoming beam is modulated by an Electro-Optic Modulator (whose functioning will be explain in the section 3.2) driven by a local oscillator. After the beam has passed through the EOM, its electric field has its phase modulated and becomes:

\[ E_{\text{inc}} = E_0 e^{i(\omega t + \beta \sin(\Omega t))} \]  

(2.10)
2.2. Pound-Drever-Hall method

This expression can be expanded, using Bessel functions, to:

\[ E_{\text{inc}} \simeq E_0 \left[ J_0(\beta) + 2i J_1(\beta) \sin(\Omega t) \right] e^{i\omega t} = E_0 \left[ J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega + \Omega) t} - J_1(\beta) e^{i(\omega - \Omega) t} \right] \] (2.11)

This holds for \( \beta \ll 1 \). This form shows the three different beams incident to the cavity: a carrier, with (angular) frequency \( \omega \), and two sidebands with frequencies \( \omega \pm \Omega \). \( \Omega \) is the phase modulation frequency and \( \beta \) is the modulation depth. If \( P_0 \equiv |E_0|^2 \) is the total power in the incident beam, then the power in the carrier is \( P_c = J_0^2(\beta) P_0 \) and the power in each first-order sideband is \( P_s = J_1^2(\beta) P_0 \).

When the modulation depth is small (\( \beta < 1 \)), almost all the power lies in the carrier and the first-order sidebands \( P_c + 2P_s \simeq P_0 \).

To express the parts reflected and transmitted by the Fabry-Pérot cavity, one can treat each beam independently and multiply each one by the reflection coefficient (or by the transmission coefficient in the case of the transmitted intensity):

\[ F_r(\omega) = r e^{i \omega \Delta \rho/\pi} - \frac{1}{1 - r^2 e^{i \omega \Delta \rho/\pi}} \] (2.12)

This leads to:

\[ E_{\text{ref}} = E_0 \left[ J_0(\beta) F_r(\omega) e^{i\omega t} + J_1(\beta) F_r(\omega + \Omega) e^{i(\omega + \Omega) t} - J_1(\beta) F_r(\omega - \Omega) e^{i(\omega - \Omega) t} \right] \] (2.13)

The reflected beam is picked off with a Polarizing Beam Splitter and a quarter-wave plate (this set up is explained in appendix D) and sent to a photodiode, which detects the power of the reflected beam \( P_{\text{ref}} = |E_{\text{ref}}|^2 \) what is after some calculations:

\[ P_{\text{ref}} = P_c |F_r(\omega)|^2 + P_s \left[ |F_r(\omega + \Omega)|^2 + |F_r(\omega - \Omega)|^2 \right] + 2\sqrt{P_c P_s} \left[ \text{Re} \left[ F_r(\omega) F_r^*(\omega + \Omega) - F_r^*(\omega) F_r(\omega - \Omega) \right] \cos(\Omega t) \right] + \left[ \text{Im} \left[ F_r(\omega) F_r^*(\omega + \Omega) - F_r^*(\omega) F_r(\omega - \Omega) \right] \sin(\Omega t) \right] + 2\Omega \text{ terms} \] (2.14)

In equation 2.14, the \( \Omega \) terms arise from the interference between the carrier and the sidebands, and the \( 2\Omega \) terms come from the sidebands interfering with each other. \( P_{\text{ref}} \) is plotted in figure 2.5 (the \( 2\Omega \) terms contain more terms in case of a higher expansion in the equation (2.10)).

The interesting terms in \( P_{\text{ref}} \) are the ones which oscillate at the modulation frequency \( \Omega \) because they carry the derivative of the reflected intensity. There are two terms in this expression: a sine
2. Frequency stabilization principle: Pound-Drever-Hall method


The Pound-Drever-Hall (PDH) method is a technique used for frequency stabilization in laser systems. It relies on the modulation of the laser's frequency and the detection of the resulting sidebands. The key principle is the use of a mixer to combine the laser's frequency modulation with a local oscillator, resulting in a product of two sine waves.

\[ P_{ref} \text{ without the time modulated part, } r = 0.9975, \Omega = 20 \text{ MHz, } L = 0.15 \text{ cm, } FSR = \frac{c}{L} = 500 \text{ MHz. In this figure the sidebands should be symmetric toward the background because of the time modulation in } P_{ref} \text{ but it was too hard to implement. An experimental } P_{ref} \text{ can be found in figure B.4.} \]

The sidebands are created by modulating the laser's frequency, and the sidebands' symmetry is influenced by the time modulation in the reference signal. The PDH method can be enhanced by using only one of the sine or cosine terms in its product, depending on the modulation frequency level relative to the cavity's response time. The modulation frequency is kept high to ensure that only one of the sine or cosine terms survives.

\[ \tau = \frac{2\pi}{F_{SR}} \]

If \( \Omega' \) is equal to \( \Omega \), as in the case of the experimental set-up I used, the \( \cos([\Omega - \Omega']t) \) term is the DC signal, which is isolated by the low-pass filter. Note that if the signal from the photodiode is not a sine but a cosine, the mixer forms the product of a cosine (from the photodiode) and a sine (from the local oscillator) and the DC signal vanishes if \( \Omega' = \Omega \). That is the purpose of the phase shifter which allows to empirically adapt the phase in one of the arm. The output of the mixer when the phases of its two inputs are not matched can produce an odd-looking error signals as shown in figure 2.6 (the error signal can also be deformed if the incoming beam is not fully well aligned in the cavity essentially because in this case the two sidebands are not reflected with the same rate).

For an appropriate phase matching, the output of the low-pass filter gives the error signal:

\[ \epsilon = -2\sqrt{P_c P_s} \text{Im}[F_r(\omega)F^*_r(\omega + \Omega) - F^*_r(\omega)F_r(\omega - \Omega)] \quad (2.15) \]

The figure 2.7 shows a plot of this error signal. \( \epsilon \) is called “error signal” since it gives the error with regard to the resonance and acts as reference for the lock.

**Remark:** The phase shift \( \Delta \) between the two arms of the mixer can be formally added for calculation in the error signal [6], which leads to:

\[ \epsilon = -2\sqrt{P_c P_s} \text{Im} \left[ (F_r(\omega)F^*_r(\omega + \Omega) - F^*_r(\omega)F_r(\omega - \Omega)) e^{i\Delta} \right] \]
Fig. 2.6: Calculated error signal for various phase shifts $\Delta$ with $r = 0.9975$, $\Omega = 20$ MHz, $FSR = 500$ MHz. The phase shift of the bottom plot is zero and increases upward in steps of $\pi/8$ until $\pi$, the different plots have offset for clarity.

Fig. 2.7: Plot of the error signal $\epsilon$, $r = 0.9975$, $\Omega = 20$ MHz, $L = 0.15$ cm, $FSR = \frac{c}{4L} = 500$ MHz.

When the carrier is near resonance and the modulation frequency is high enough we can assume that the sidebands are totally reflected, $F_r(\omega \pm \Omega) \simeq 1$, then:

$$F_r(\omega)F^*_r(\omega + \Omega) - F^*_r(\omega)F_r(\omega - \Omega) \simeq -2iIm[F_r(\omega)]$$

Since we are near resonance, one can write:

$$\frac{\omega}{FSR} = 2\pi n + \frac{\delta\omega}{FSR}, \quad n \in \mathbb{N}$$

where $\delta\omega$ is the deviation of the laser frequency from resonance. If one assumes that the cavity has a high Finesse then $\delta \simeq \frac{\pi}{1 - r^2}$ and the reflection coefficient becomes:
2. Frequency stabilization principle: Pound-Drever-Hall method

\[ F_r \simeq \frac{i \delta \omega}{\pi \Delta \nu} \]

where \( \Delta \nu \equiv \frac{FSR}{8} \) is the cavity linewidth. The error signal is then proportional to \( \delta \omega \), and this approximation is good as long as \( \delta \omega \ll \Delta \nu \).

Then the error signal can be expressed as:

\[ \epsilon \simeq -\frac{4}{\pi} \sqrt{P_s P_c} \frac{\delta \omega}{\Delta \nu} \]  \hspace{1cm} (2.16)

The linearity of the error signal near resonance allows one to use standard tools of control theory to stabilize the frequency of the laser. For this purpose the error signal from the low-pass filter is sent through a servo amplifier and into the tuning port of the laser, locking the laser to the cavity (the stabilization principle will be explained in the next chapter). It will be useful to express the error signal in terms of the regular frequency \( f = \frac{\omega}{2\pi} \), instead of \( \omega \), and define the slope of the error signal at resonance:

\[ \epsilon = D \cdot \delta f \] \hspace{1cm} (2.17)

This slope will be used to calculate the laser linewidth and can be expressed in a way more convenient for measurements:

\[ D = \frac{2 \cdot \text{error signal peak} - \text{to} - \text{peak amplitude}}{\Delta \nu} \] \hspace{1cm} (2.18)

One can see from equation (2.17) that the steeper the slope of the error signal is, the higher is the sensitivity to fluctuations of the laser frequency. One way could be to increase the linewidth \( \Delta \nu \) of the cavity but this parameter can’t be decreased arbitrarily since it depends essentially on the cavity Finesse and then on the quality of the mirrors. Another more immediate way is to optimize the modulation depth \( \beta \) to get a maximal value for \( \sqrt{P_s P_c} \). If this expression is written in terms of Bessel functions a maximum can be found at:

\[ \frac{P_s}{P_c} = 0.42 \]

resulting in an optimal modulation depth \( \beta = 1.08^1 \). \( \beta \) depends on the action of the EOM on the beam and with the equations leading to the EOM (which will be explained in section 3.2) one can find the optimal current which has to be applied on the EOM to reach the optimal modulation depth. In practise the amplitude of the modulation applied on the EOM is just empirically increased until the error signal amplitude becomes maximum.

Once one gets an error signal, it can be used to stabilize the laser frequency what is the subject of the following chapter.

\(^1\) With \( \beta \) close to one, higher sidebands can’t be neglected and contribute to the error signal. Nevertheless it can be shown that they do not change the slope at the center and therefore do not decrease the sensitivity.
3. STABILIZATION OF THE 732 NM WITH THE FABRY-PEROT CAVITY

3.1 Optical and electronic set-up

The set-up implemented for the frequency stabilization of the 732 nm is the presented in figure 3.1. The optical part is almost the one used for the basic PDH method explained in section 2.2. The two lenses before and after the EOM are used to focus the beam in the EOM crystal to improve the frequency modulation and also to reduce the transverse size of the beam (telescope configuration). As explained in the chapter 1 the power of the beam used for the stabilization is very low (about 8 mW) and it decreases to about 40 µW in front of the cavity because of the losses. This induces that the coupling with the cavity has to be very optimal (so the 10 cm lens in front of the cavity is really essential now) to have enough power on the photodiode and a good error signal. Thus the 5 cm focal-length lenses in front of the photodiodes on the back of the cavity and on the output are essential for the 732 nm beam to collect all the light possible.

Concerning the electronic part, only the three modules essential for the stabilization have been represented. The power supply of the laser also uses two modules for the temperature stabilization of the taped amplifier and the laser diode and two modules for the current regulation of the taped amplifier and the laser diode. The three modules are:

- the Scan control module contains two potentiometers: one for the offset of the piezo and the other for the amplitude of the grating motion around the position set by the offset (piezo amplitude). It is important to remark that when the amplitude of the scan decreases, it also decreases the range of the scan and then the number of modes which can be visualized on the oscilloscope during one single trip. This creates a zoom effect which stretches the signal. One pen-potentiometer (this kind of small potentiometer is called trimpot) for the feed forward is also present, its utility has been explained in the section in the chapter 1. There are also two outputs: the signal for the piezo and a trigger for the oscilloscope.

- the PDH module includes the mixer and the low-pass filter used in the basic PDH set-up to create the error signal from the signal of the photodiode (so one input for the signal from the photodiode and one output for the error signal monitoring). This module also provides the modulation signal for the EOM, so it displays an output for this signal with an amplitude potentiometer (and an internal connection to the mixer of the PDH calculator). Two potentiometers allow one to set the phase shift to obtain a symmetric error signal and the gain for the amplitude of the error signal. An extra phase shifter has been added after the photodiode because the phase shift of the PDH module was not sufficient to get a good error signal.

- the PID (Proportional-Integrative-Derivative) module is the part in charge of the locking of the piezo so it needs the error signal. For the stabilization circuits three trimpots plus an overall gain allows to adjust the value of the three parts of the regulator. A switch allows one to choose to lock on a positive or negative slope of the error signal and of course a monitor output. The principle of the PID and the set point trimpot utility will be explained in the section 3.3.

One can note that the error signal is also sent to a FET input on the laser box, this will be explained in the section 3.3.

With this set-up a transmitted intensity with an amplitude of 8 mV and an error signal of about 300 mV have been obtained.
3. Stabilization of the 732 nm with the Fabry-Perot cavity

![Diagram of the stabilization setup](image)

Fig. 3.1: Electrical and optical set-up used to stabilize the laser frequency. Solid colored lines are optical paths and dashed lines are electric signals (the internal connection is an electric signal).
3.2 Frequency modulation : Electro-optic Modulator

An essential part of the PDH method is the modulation of the beam by the EOM and a part of my internship was to implement its power supply. The functioning of this device is exposed in the two following sections.

3.2.1 Functioning

An EOM is a crystal whose refractive index changes when an electric field is applied on it (this effect belongs to the non-linear optical effects) [2]. Indeed the refractive index can be expanded with the following manner (Taylor series):

\[ n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \ldots \]

where \( a_1 = \left( \frac{dn}{dE} \right)_{E=0} \) and \( a_2 = \left( \frac{d^2 n}{dE^2} \right)_{E=0} \)

And with the conventional notation: \( r = -\frac{2a_1}{n^3} \) and \( s = -\frac{a_2}{n^3} \), \( n(E) \) becomes:

\[ n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s n^3 E^2 \]

In the case of an EOM the interesting effect can be limited to the linear dependance of \( n(E) \) with \( E \) called Pockels effect, that is:

\[ n(E) \simeq n - \frac{1}{2} r n^3 E \]

The set-up for phase modulation with an EOM is presented in the figure 3.2.

![Fig. 3.2: Basic set-up of a tension applied to an EOM, the top of the crystal is coated with gold and the bottom is linked to ground. The EOM then behaves as a capacitor (the crystal is non conductive).](image)

The phase shift induced by the EOM on the beam is \( \phi = n(E)k_0L = 2\pi n(E)\frac{L}{\lambda_0} \), what leads to:

\[ \phi \simeq \phi_0 - \pi \frac{V(t)}{V_\pi} \]  \( \text{where} \ \phi_0 = \frac{2\pi nL}{\lambda_0} \) and \( V_\pi = \frac{d\lambda_0}{Lrn^3} \)

With an incident beam of \( E_{\text{inc}} = E_0e^{i\omega t} \), after the EOM it will become with an applied tension of \( V(t) = V_0\sin(\Omega t) \):

\[ E_{\text{out}} = E_0'e^{i(\omega t+\phi)} = E_0'e^{i[\omega t+\beta \sin(\Omega t)]} \]  \( \text{(3.1)} \)

where \( E_0' = E_0e^{i\phi_0} \) and \( \beta = -\pi \frac{V_0Lrn^3}{d\lambda_0} \) (\( r < 0 \) so \( \beta > 0 \)). The equation (3.1) proves the hypothesis done in the section 2.2 about the action of the EOM on the beam.

The EOM used in this work is a LiNbO\(_3\) crystal whose characteristics are (from [10]): \( r = 30.8 \) pm/V, \( n = 2.30 \) at 22 \(^0\)C, \( L = 3 \) cm, \( d = 3 \) mm, what leads to:

\[ \beta \simeq 0.0108 V_0 \]  \( \text{(3.2)} \)
3. Stabilization of the 732 nm with the Fabry-Perot cavity

3.2 Power supply for the EOM

The equation (3.2) shows that to reach the optimum of modulation defined in the section 2.2 ($\beta = 1.08$) one has to apply a very high tension of at least 100 Volts on the EOM.

The figure 3.3 shows the electrical set-up I developed to supply the EOM crystal. The original set-up was only composed of the local oscillator and the Minicircuits amplifier but it was not enough to obtain a sufficient amplitude for the error signal. We then decided to realize a transformer to form an independent resonant LC circuit with the EOM and the secondary coil. The transformer ensures the isolation from the primary circuit and thus a very low resistance in the secondary circuit. Moreover, as the amplification from one coil to another in a transformer depends basically on the ratio between the number of turns in the two coils (with the homemade transformer this leads to a ratio of 31.5:2 for the secondary coil). Then the secondary coil receives about 15 times the power applied on the primary coil and the transformer provides an extra amplification.

As seen in the previous section the EOM acts as a capacitor of about $C = 44$ pF (with a short cable), as the frequency modulation from the local oscillator is 19.82 MHz the inductance has to be tuned to about $L = 2.2 \mu$F such that the resonant frequency of the LC circuit equals the local oscillator one. Experimentally, the power in the transformer is very high and dissipates in the coils and essentially in the secondary coil. Hence, the wire with which is made the coil must not be too thin otherwise the coil will heat too much and then be very instable because of the dilatation. After some burned coils, a very stable secondary coil has been build: $n = 31.5$ turns of a $r = 0.8$ mm diameter coated wire around a strong plastic pipe with a diameter of $d = 6.2$ mm (theoretical inductance calculated with the formula $L = (d^2 n^2) / (nd + 0.45d)$), what assures a very stable coil with a temperature of about 31 °C in the box.

Another problem was to adapt the primary coil in order to keep the secondary circuit resonant with the oscillator frequency but with a resistive impedance of at least 40 Ω on the input of the primary (Minicircuits amplifier needs to be able to discharge itself). The complex impedance has been measured with an impedance analyzer plugged on the primary coil, the real part $R$ of the impedance telling the effective resistivity seen by the amplifier and the imaginary part $X$ telling the resonant matching of the LC circuits with the applied 19.82 MHz (perfect resonance for $X = 0$). The analyzer also measures the Standing Wave Ratio which is an other variable characterizing the matching (SWR must be less than 2 for a perfect matching).

This matching was achieved mainly by modifying the number of turns of the primary coil and by moving it along the secondary coil. The last problem was that the transformer had to be locked in a metallic box (Faraday cage) to avoid any noise radiation on the optical table and then the metallic box creates a capacitive coupling modifying the resonance frequency, so do the screws used to close...
3.3 Frequency lock

Finally, a good compromise was found with characteristics of \( R = 108, \ X = -2, \ SWR = 2.3 \) for \( f = 19.82 \) MHz and almost the same range of values for frequencies from 19.8 to 19.85 MHz (because the frequency of the local oscillator is not very stable). But the real check of the efficiency of this set-up was the shape of the error signal and in the figure 3.6 one can see second order sidebands which are a proof of a high \( \beta \). Moreover by changing the amplitude of the modulation signal (on the PDH module) it was possible to find a value with which the error signal was maximal. Then it was empirically set to a value for the amplitude corresponding to \( \beta = 1.08 \).

The stability of the modulation and thus the stability of the transformer were evaluated in applying the optimum power during long period without any discernable change in the error signal or overheating.

3.3 Frequency lock

The PDH modules allows one to use the error signal to stabilize the length of the extra and the laser frequency cavity. The process is the following : the scan of the piezo is turned to zero with a proper offset in order to adjust the length of the extra cavity around the peak of a resonance, more precisely at the zero crossing of the slope of the error signal. Then the signal seen by the PID regulator is resumed to a linear slope symmetric around the zero crossing.

The piezo regulator is made up of a PID (Proportional-Integrative-Derivative) controller. Let us suppose that the grating is exactly on the right position described above and that a perturbation lightly moves the grating on the right of the resonance namely on the positive part of the error signal slope. The PID controller detects this excursion by comparing a set-point level (usually set a bit above the zero of the error signal with a trimpot of the PID module) with the zero of the error signal). The response of the controller can then be decomposed in three actions :

- the P-part sends a signal proportional to the deviation on the piezo to go back to the zero-crossing,
- the I-part integrates the deviation with respect to the time to measure if the deviation is above or below the zero-crossing.
- the D-part has almost the same role than the I-part but provides a faster response. Indeed the derivative of the excursion on the error signal immediately gives the sign of this excursion (positive if the piezo goes up on the slope or negative if it goes down), whereas the Integrative part, while giving the side of the excursion, also acts like a memory of previous variations.

When the PID controller is in this sequence of back-action the output monitor will show an error signal which looks like noise and is in fact a recollection of round-trip around the zero-crossing on the slope of the error signal out of lock.

Figure 3.4 shows the difference between the error signal and the transmitted intensity in and out lock. The appearance of the inloop error signal (in red behind the usual error signal) is the one explained in the previous paragraph. The amplitude of this signal has to be compared to the amplitude of the slope of the error signal since the regulation by the piezo controller is done along this slope. The smaller this amplitude is, the smaller the excursion of the piezo from the resonance will be since this amplitude is related to the amplitude of the piezo oscillation (the coefficient between the two is the value of the slope of the error signal). Another observation can be done to evaluate the lock efficiency: when the laser is locked the level of the transmitted intensity should exactly be at the maximum of the transmitted intensity when the laser is unlocked, what means that the extra-cavity length almost matches with the top of the resonance (see figure B.3 for more clarity).

On the figure 3.4 one can see that the average amplitude of the error signal when the PID regulator is working at half the amplitude of the slope out of lock what means that the excursions of the laser frequency are very small and thus that the stabilization is very efficient. The stability of the lock can also be evaluated in softly hitting the laser box or the optical table and see if the stabilization holds on and the frequency stays on the same mode or on the contrary if the laser jumps to an other mode
3. Stabilization of the 732 nm with the Fabry-Perot cavity

Fig. 3.4: Error signal in (in red) and out of lock (in blue) namely when the PID module is regulating or not.

(this method is more used to check the stability while the settings of the regulator parameters with the three trimpots dedicated to the P, I and D parts). The most accurate way of checking the lock stability simply remains to let the laser locked during long periods and to check with a wavemeter if the frequency changes.

In term of stabilization, the frequency of the piezo scan is at the maximum 2 or 3 kHz what means that all the perturbations whose frequencies are above this range can not be regulated by the piezo. That is why an extra stabilization on the courant of the laser diode is implemented via a Field Effect Transistor (FET). The functioning of this stabilization is more or less the same as the one of the piezo stabilization except that the frequency range of the FET is around 650 kHz and that it is always working. The FET regulation has a very visual effect on the slope of the error signal as shown on the figure 3.5. The slope of the error signal seems to be broadened by some noise which is in fact the oscillation of the FET regulation along the slope of the error signal exactly like the error signal of the cavity when it is locked. It has to be noticed that the FET adds an other parameter to set during the settings of the regulator.

Fig. 3.5: On the left side the error signal without FET regulation and on the right side with FET regulation.

In all the previous writing a huge source of destabilization has been ignored. Indeed the PDH method used for stabilizing the laser frequency assumes that the Fabry-Pérot cavity length always remains the same. This is absolutely not guaranteed since any perturbation like thermal expansion of the materials, mechanical vibrations or strong acoustic noise can slightly displace the mirrors and change the cavity length. The resonance frequency of the cavity would then change as well as the error signal. This perturbation would wrongly be interpreted by the PID controller as an excursion of the laser frequency and induce a wrong regulation. To overcome this problem the cavity length is also stabilized by PDH method with an extra laser. The necessary set-up additionally allows one to measure the cavity linewidth which is needed for the measurement of the laser linewidth described in the following section. The cavity length stabilization is presented in appendix B.
The light of the laser is constantly sent to a wavemeter which measures the wavelength with an accuracy of $10^{-5}$ nm. This allows to check the frequency stabilization performance of the laser:

- When neither the Fabry-Pérot cavity length nor the laser frequency are locked, the stability is of $10^{-3}$ nm.
- When the Fabry-Pérot cavity length is not locked and the laser frequency is locked, or the vice-versa, the stability approximates $10^{-4}$ nm.
- When both are locked the stability is of $10^{-5}$ nm.

### 3.4 Signals and laser linewidth

The measurement of the laser linewidth presented in the following provides for a quantitative characterization of the laser lock. If we assume that the frequency distribution of the locked laser has a lorentzian shape centered on the frequency measured by the wavemeter then the linewidth is the FWHM as also the average frequency excursion of the laser around the centered frequency.

#### 3.4.1 Laser linewidth

When the laser is locked, the error signal is a good indicator for the quality of the lock to the cavity. However, it is not completely straightforward to determine the laser-linewidth from the error signal. With the help of the autocorrelation function one is able to calculate it. In the following we consider a fictitious situation, namely an effective broadening of the lasing spectrum, to present how the laser linewidth can be deduce from the error signal. Only basic ideas are introduced, more details can be found in [7, 9].

Usually, one deduces the power spectrum of the laser light, starting from an infinitely narrow spectrum. This spectrum is broadened with an AOM driven by a voltage controlled oscillator (VCO) (the functioning of the AOM is explained in the section 4.1). The VCO is fed by a noise source with a Gaussian probability distribution in a certain bandwidth. The output signal of the oscillator is of the form:

$$V_f(t) = A_0 e^{i\omega_0 t + \Phi(t)}$$

(3.3)

where

$$\Phi(t) = 2\pi \int_0^t DV(t')dt'$$

(3.4)

where one assumes that $\Phi(t)$ is a Gaussian process since $V(t')$ is a Gaussian process. The slope of the tuning curve of the VCO is denoted with $D$ (units Hz/V) and $\omega_0$ is the central frequency. After some calculations the power spectrum of the laser is:

$$W_f(\Delta\omega) = \frac{A_0^2}{2\pi} \int_0^\infty d\tau \cos(\Delta\tau) \exp[-8(\pi D)^2 \int_0^\infty W_v(\omega') \left( \frac{\sin(\omega' \tau/2)}{\omega'} \right)^2 d\omega']$$

(3.5)

where $\Delta\omega = \omega - \omega_0$, $\omega$ is the frequency used to express the frequency distribution of $W_f$, $W_v(\omega')$ is the power spectrum of the noise. The main idea is now to consider the case of a rectangular input noise power spectrum with a cut-off at frequency $B$ and an amplitude of $V_{\text{rms}}^2$,

$$W_v(\omega) = \begin{cases} V_{\text{rms}}^2, & 0 < \omega < 2\pi B \\ 0, & \omega > 2\pi B \end{cases}$$

(3.6)

In the case of a well stabilized laser the ratio between deviation frequency to cutoff frequency $\frac{DV_{\text{rms}}}{B}$ is small, then power spectrum can be calculated and it reads:

$$W_f(\Delta\omega) = \frac{A_0^2}{2\pi} (\pi^2 D^2 V_{\text{rms}}^2 / B) V_{\text{rms}}^2 / B + (\Delta\omega)^2$$

(3.7)
which is a lorentzian distribution with a FWHM of:

\[ FWHM = \frac{\pi^2 D^2 V_{\text{rms}}^2}{B} \]  

(3.8)

This equation can be translated to our set-up. The error signal when the cavity is locked (see figure 3.4) acts as the noise source and as the piezo in the laser changes the frequency of the laser when it moves the grating it can be considered as the AOM which changes the frequency of the laser in the article [7]. Then the slope of the VCO will be replaced by the inverse of the slope of the PDH error signal, the inverse because the slope of the VCO is the link between Hz in Volt and the slope of the error signal is the link between Hz (or time on the oscilloscope) and Volt. As the inloop error signal acts like the noise, one has to do the FFT of this signal to measured the \( B \) and \( V_{\text{rms}} \) parameters. The inverse of the error signal will be measured using the equation (2.18) what leads to:

\[ FWHM = \frac{\pi^2 V_{\text{rms}}^2 \Delta \nu^2}{4(\text{peak to peak amplitude})^2 B} \]

(3.9)

3.4.2 Results

So to measure the laser linewidth one needs the slope of the error signal, which is proportional to the peak to peak amplitude of the error signal and the active linewidth of the cavity (active because the measured is done when the cavity is stabilized to a reference laser).

As explained in the section 3.1 the main problem of the linewidth measurement is that the width of the error signal directly depends on the amplitude of the scan so one needs a scale between time on the oscilloscope and frequency. Fortunately as seen in the section 2.2, the frequency distance between the two sidebands is twice the modulation frequency (that is two times 19.82 MHz here) so by measuring the time between the two zero-crossings of the sidebands (\( \Delta S \) on the figure 3.6) and the time between the two maxima of the error signal one can obtain the time/frequency scale. The principle of these measurements are shown on the figure 3.6.

Another problem is that the FET regulation makes the error slope very noisy (as shown on the figure 3.5) what makes the measurement of the cavity linewidth really inaccurate. Indeed the measurements done give in average a cavity linewidth of 5.4 ± 0.2 MHz (the error range of the linewidth is far underestimated), this value has to be compared to the passive value of the linewidth of 4.25 ± 0.11 MHz measured in the appendix B.3. The problem is that the active linewidth should be smaller than the passive one since the cavity length is stabilized. So for the following calculations the passive cavity linewidth will be used since it comes from the most accurate measurements.

The theory used for the linewidth measurement requires that the noise frequency distribution ie in our case the inloop signal frequency distribution is a square distribution as described in the equation (3.6). One can see on the figure 3.7 that the FFT of the inloop signal does not exactly present a square shape even if a quite strong shoulder can be distinguish after the peak at about 600 kHz (this peak will be explained later). That is why measurements have been done from the top of the peak to the bottom of the ramp. The noise at -85 dBm (for frequencies superior than \( B_\gamma \)) is taken as the zero indeed -85 dBm with an impedance of 1 MΩ (what was the case of the oscilloscope) corresponds to 2 mV so in all the measurements of \( V_{\text{rms}} \), 2 mV have been subtracted. The value of \( V_{\text{rms}} \) has been measured by the oscilloscope with an average and an integration of the signal from 0 to the cut-off frequency \( B \) chosen and then converted in Volts. Two series of measurements have been done on two different inloop signals.

**Remark:** As the measurements are essentially about power and amplitude it is important to realize them with as less electronic modifications (like gain, offset,...) as possible. In our case the measurements have been done between the PDH and PID modules to get an error signal while avoiding PID electronics.
Fig. 3.6: Measurements of the characteristics of the error signal of the 732nm laser.

![Graph showing error signal characteristics](image)

Fig. 3.7: FFT of the inloop signal when the cavity is locked. The red square distribution is an approximation of the shoulder of the FFT signal. The four yellow markers indicate the four different cut-off frequencies chosen as references for the measurements of $V_{\text{rms}}$.

![Graph showing FFT of inloop signal](image)

The results are shown in the charts 3.8. As the square distribution is a coarse approximation of the FFT signal, the final value will be an average along the slope (and then on the values $\beta$, $\gamma$ and $\delta$). This leads to:

$$FWHM = 69 \pm 41 \text{ kHz for the mean values},$$

and $$FWHM = 41 \pm 28 \text{ kHz for the area values},$$

for the right chart (the left chart can hardly be used since the $\beta$ value is missing).

These results are a bit too good since the expected value is about 100-200 kHz. Indeed the stabilization chain explained in the appendix C induces that each successive steps in the locking chain broaden more and more the linewidth of the stabilized lasers (see figure C.1). This induces that the linewidth of the 732 nm laser depends on the stability of its cavity, which stability depends on the...
stability of the 793 nm laser, which stability depends on the stability of its own cavity which stability depends on the stability of the 852 nm laser with a stability which goes worse at each step. The 732 nm laser linewidth should then be larger than the one of the 793 nm laser and yet this linewidth was measured with accuracy at 125 kHz. This incoherence between the two linewidths can be explained since the method used in this report for measuring the laser linewidth at the point which corresponds to FET frequency components (explanations below) and therefore will not be used for final results.

- The spectroscopy of the trapped Ca$^+$ ion i.e. the study of the characteristics of the 732 nm transition.

- A beat measurement of the laser with another 732 nm laser whose linewidth is known. The study of the interferences between the 2 lasers gives the unknown linewidth (see [9]).

These two methods were too complex to be implemented during my stay.

The value of the FWHM can be seen as the frequency excursion around the central light frequency which is about 409 THz for the $4S_{1/2} - 3D_{3/2}$ transition. The excursion is then less than $10^{-7}$ %. This efficiency of the stabilization will be compared to the objectives in the conclusion.

It has to be noticed that some groups have managed to stabilize lasers to sub-Hertz linewidth [14] and it is very usual to obtain FWHM of a few Hertz with high finesse cavities (see [9]).

Finally, let us note that the FFT of the error signal when the laser is locked also allows one to visualize the two parts of the stabilization. In figure 3.9 the peak at 600 kHz corresponds to the FET regulator, this peak can be decreased by turning down the error signal gain on the PDH module (figure 3.1). At low frequencies one can see the frequency components of the piezo regulation, about a few kHz. These two regulations allow one to stabilize a large set of perturbations.
Fig. 3.9: The two parts of the stabilization. FET regulator for high frequencies about 600 kHz and piezo regulation for low frequencies about 3-4 kHz.
3. Stabilization of the 732 nm with the Fabry-Perot cavity
4. ADJUSTMENT OF THE LASER FREQUENCY

When the cavity and the laser are locked the laser wavelength can be set at 732.389 nm which is the reference value for the $4S_{1/2} - 3D_{3/2}$ transition. To optimize the interaction between the ion and the laser beam one needs to be able to adjust more precisely the wavelength (at $10^{-5}$ nm what is the accuracy of the available wavemeter) without disturbing the lock. This can be achieved using an AOM set-up. The Acousto-Optic Modulator functioning is explained in this section.

4.1 Frequency shift : Acousto-Optic Modulator

A radio-frequency signal applied to a piezo-electric transducer (see figure 4.1), bonded to a suitable crystal, will generate an acoustic wave (angular frequency $\Omega$ and wave vector $\vec{Q} = q \vec{u}_x$, $q = \frac{\Omega}{c}$). This acts like a phase grating, traveling through the crystal at the acoustic velocity $v$ of the material [2]. This induces a modulation of the molecular density and then a modulation of the refractive index: $n(x) = n_0 + \Delta n \cos(qx - \Omega t + \Phi)$. If one considers an incident optical wave with an angular frequency $\omega$:

$$S_{in}(x, z, t) = S_0 e^{i(kz - \omega t)} , k = \frac{2\pi}{\omega}$$

and the phase shift induced by the phase grating:

$$\phi(x, t) = \phi_0, \cos(qx - \Omega t + \Phi),$$

then the output field reads:

$$S_{out}(x, z, t) = S_0 e^{i(\phi(x, t) + k z - \omega t)}$$

which can be expanded into Bessel functions:

$$S_{out} \propto \left[ J_0(\phi_0) + iJ_1(\phi_0) e^{i(qx - \Omega t)} + iJ_1(\phi_0) e^{-i(qx - \Omega t)} + \ldots \right] e^{i(kz - \omega t)} \quad (4.1)$$

This expression shows that the incident beam is refracted in different waves with angular frequencies:

$$\omega_n = \omega \pm n \Omega , \quad k_n = \frac{\omega_n}{c} , \quad n \in \mathbb{N} \quad (4.2)$$

Moreover each wave has to satisfy the momentum conservation of the wave vectors so each wave goes out with an angle $\theta_n$ from $\vec{u}_z$ verifying (as $\Omega \ll \omega$ we have $k_n \approx k$):

$$\sin \theta_n = \pm n \frac{\Omega}{\omega} , \quad n \in \mathbb{N} \quad (4.3)$$

The corresponding diffraction pattern is shown on the left part in figure 4.1.

It can be shown that for a certain value of the incident angle most of the power goes out in the 1st order (or in the -1st it depends on the set-up). This set-up is called the Bragg diffraction (left part of figure 4.1) with an incident angle of:

$$\sin \theta_B = \frac{\Omega}{2\omega}$$

In practical work this angle is empirically set by tilting the AOM and measuring the power of the first order. It has also to be noticed that the amplitude of the RF sinusoidal signal applied on the AOM modifies the scattering of the beam and the efficiency of the Bragg diffraction.
4. Adjustment of the laser frequency

![Diagram of diffraction induced by the AOM.](image)

**Fig. 4.1:** Diffraction induced by the AOM. The right figure shows the diffraction with Bragg condition.

### 4.2 Optical and electronic set-up

![Diagram of optical and electronic set-up for the AOM.](image)

**Fig. 4.2:** Optical and electronic set-up for the AOM. Solid colored lines are optical paths and dashed lines are electric signals.

Figure 4.2 shows the set-up for the AOM, the optical frequency is shifted twice making use of back reflection what leads to $\omega - 2\Omega$ on the input of the optical fiber and toward the trapped ion. It is important to say that the AOM does not affect the laser linewidth. The loss induced by the PBS the first time the laser beam goes through it is sent to a wavemeter to measure continuously the wavelength and check the stability of the lock (it has to be noticed that the measured frequency is then $\omega$). The two 20 cm lenses are used to focus the beam on the AOM and optimize the diffraction. The pinholes serve to block the $0^{\text{th}}$ orders. The AOM and the arm after it are tilted to match the Bragg angle in both single and back trips.

The AOM set-up is thus very useful but it induces some light losses, indeed in front of the optical fiber and after a lot of optimization there is 45-50% of the power from the laser box (500 mW). The coupling in the optical fiber also causes some losses, at the end of the fiber there is about 80% of the initial power available for the interaction with the ion.

By changing $\Omega$ one can adjust the wavelength with an accuracy of $10^{-5}$ nm to maximize the interaction with the ion’s $4S_{1/2} - 3D_{3/2}$ transition. This fine setting of the frequency is experimentally performed by sending the light to the trapped ion in order to do a spectroscopy of the $4S_{1/2} - 3D_{3/2}$ transition. This has not been done since the ion trap was used for another experiment and the necessary optical devices (like lenses for tight focusing) would have been too long to set up.

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1 The AOM set-up can also be used to create laser pulses and switch on and off the interaction with the ion (this is only switching on and off the RF signal and thus the scattering) what is essential to implement ionization, cooling, state detection,... sequences.
5. FINAL RESULTS AND CONCLUSION

Thanks to the setup explained in this report three laser characteristics necessary to drive the $4^2S_{1/2}-3^2D_{3/2}$ transition have been improved:

- a laser power of about 200 mW to excite the ion,
- a centered frequency stable at $10^{-5}$ nm during hours and tunable on the exact $4^2S_{1/2}-3^2D_{3/2}$ transition wavelength,
- a narrow linewidth of about 60 kHz (which has to be compared with the initial linewidth of a few MHz).

It was not possible to test the efficiency of this light on the $^{40}\text{Ca}^+$ ion but we can nevertheless compare these characteristics with the values used in the experiment described in the article [13]:

- power of 1.8 mW,
- spot size of 14 µm on the ion,
- linewidth of about 20 Hz.

These values leads to a measured Rabi frequency (which characterizes the coupling laser/ion) of about 110 kHz which is a good benchmark for the implemented stabilization. With the characteristics of the stabilized laser we lose a factor 3000 with the linewidth but we gain a factor 100 with the power and 10 with the focusing (the laser beams are focused to the trap with a high numerical aperture lens what leads to a spot size of a few µm on the ion). So we should theoretically obtain a Rabi frequency of the same order of magnitude and we can say that the first step of the 732 nm laser is achieved.

Nevertheless the inaccuracy of the linewidth measurements moderates the conclusion, indeed the value of 60 kHz is certainly wrong for the reasons explained in the section 3.4.2. The best proof of the efficiency remains the spectroscopy of the $4^2S_{1/2}-3^2D_{3/2}$ transition.

Concerning the future works on my setup, the final project is to add a high finesse cavity to the setup I worked on to gain a better linewidth while using the homemade cavity for coarse stabilization.

My internship was particularly rewarding since I was in charge of a whole independent setup in all its aspects: electronic, computer, optics, mechanics, theory, etc. It has really opened my mind to experimental physics and especially to quantum optics.
Thanks :

I would like to thank the ICFO, all the “Quantum information and quantum optics with single trapped atoms” group and especially Juergen for giving me the opportunity to effectuate a so interesting internship in such a good atmosphere. I also warmly thank Albrecht, Matteo, Tristan, Hannes, Nico, Carsten, Marc, Felix, Daniel, Jan, Francois, Ricardo and Jose-Carlos for all their help.
It was a real pleasure to work with all of you !
A. OVERVIEW OF THE OPTICAL SET-UP

Figure A.1 presents the global optical set-up implemented for the frequency stabilization of the 732 nm laser. This setup can be decomposed in four independent parts:

- The laser itself. It emits two beams: one powerful beam which will be sent to the ion and a weaker one used for the laser stabilization.

- The cavity stabilization setup which uses an independent 793 nm laser beam to stabilize the cavity length.

- The laser frequency stabilization setup whose key component is an Electro-Optic Modulator.

- The output setup which allows one to adjust the laser frequency (independently from the laser itself) thanks to an Acousto-Optic Modulator, to measure the wavelength and to sent the light to the ion.

The electronic setup has not been represented in the figure A, mainly for clarity reasons. It basically consists of the laser power supply, different power supplies and signal generators for the photodiodes, AOM, EOM, amplifiers,... and the cavity locker device used for cavity stabilization.
Fig. A.1: Global optical set-up.
B. STABILIZATION OF THE FABRY-PÉROT CAVITY WITH A 793 NM LASER

In this section, we describe the PDH method implemented to stabilize a Fabry-Pérot cavity to a laser beam. Basic ideas and equations remain exactly the same than with the stabilization of the laser frequency onto a cavity. In this case the frequency stability is replaced by the stability of the length of the cavity and the reference is not the cavity but the laser. The length has to be stabilized because of length variation caused by temperature changes and acoustic vibrations. This means that the 793 nm laser frequency has also to be stabilized to act as a reference what leads to the global locking chain explained in the section C.

B.1 Overview of the cavity stabilization set-up

The figure B.2 shows the optical and electronic set-up used for the stabilization. In this work the cavity is stabilized with a 793 nm laser beam, which is already modulated in current (which is basically the same than the phase modulation used in the PDH method explained in the section 2.2) with a modulation frequency of 20 MHz. The optical set-up is the one used for the laser stabilization.

\[
\frac{3}{\pi} = \frac{r}{1 - r^2} \simeq 626.75 , \quad FSR = \frac{c}{4L} \simeq 500 \text{ MHz} , \quad \Delta \nu = \frac{FSR}{3} \simeq 800 \text{ kHz}
\]

Fig. B.1: Schema of the homemade Fabry-Pérot cavity used in the set-up described in the figure B.2.

As in this case the cavity is the device which has to be stabilized it owns a tuneable degree of freedom which is its length. The figure B.1 presents a cross-section of this cavity with the main elements in it. The cavity is composed of two identical dichroic spherical mirrors with a reflectivity of 0.9975 and a radius of curvature of 15 cm. As this cavity is built to be a confocal cavity the length between the mirrors is about 15 cm as seen in the section 2.1.1. The back mirror can be moved along the optical axis by three piezo-electric ceramics. Both mirrors are fixed to an aluminium tube. To avoid thermal expansion and length variation of this tube the temperature of the cavity is isolated from the outside and stabilized by a thermometer and a heating wire. Then there are three BNC connectors linked to the cavity locker (see figure B.2).

These characteristics of the cavity allows one to calculate the theoretical values for the parameters described in the section 2.1.2:
B. Stabilization of the Fabry-Pérot cavity with a 793 nm laser

Fig. B.2: Set-up used in this work to stabilize the cavity length. One can remark that the error signal slope does not exactly match the peak of the transmitted signal as it should be, this time delay is mainly caused by the time response of the photodiodes and the electronic delays in the cavity locker. Solid colored lines are optical paths and dashed lines are electric signals (the internal connection is an electric signal).
The electronic device used to stabilize both cavity length and temperature was designed in the laboratory. It can be separated in three different parts.

- The left part (figure B.2) stabilizes the temperature of the cavity with respect to a tuneable set point (so it needs one input for the cavity temperature and one output for the heater). The cavity temperature is stabilized at 37.05 ± 0.01 °C.

- The middle part is the scan control of the piezo. As explained in the section 2.1.1 and directly shown by the equation (2.3) when the cavity length is varied it allows one to scan over the different resonances. Two potentiometers allow one to set the offset of the scan (basically the length starting point of the oscillation of the mirror) and the amplitude of the scan (the amplitude of the oscillation and then the amplitude of the length variation). The basics of the cavity length scan is the same than for the Scan module of the laser (see section 3.1) : when the amplitude of the scan decreases, it creates a zoom effect which stretches the signal.

- The right part of the cavity locker contains the Pound-Drever-Hall module and the piezo controller (they are equivalent to the laser modules). The modulation frequency (20 MHz) for the mixer comes directly from the local oscillator of the 793 nm laser. Three potentiometers allow to set the phase shift to obtain a symmetric error signal, the offset and the gain for the amplitude of the error signal. These two settings are essentially used to set the correct shape of the error signal needed to lock the cavity.

It has to be noticed that world-wide experiments rather use High Finesse cavity build with ultra low expansion material and high quality mirrors set up in vacuum chambers. The reached finesse are usually more than 100 000 (see [9]) what leads to a cavity linewidth lower than 1 kHz. Moreover the length of this kind of cavity does not need to be permanently stabilized since the quality of the material, the vacuum chamber and a vibration isolation ensure that the cavity is almost insensible to temperature or vibration perturbations (for more explanations about this kind of cavity see [9]). Despite these advantages the choice was made to use the homemade cavity described previously since it is easier and faster to set up.

**B.2 Cavity lock**

The right part of the cavity locker allows one to use the error signal to stabilize the length of the cavity. The process is exactly the same as the one used to stabilized the laser frequency (explained in section 3.3). The only difference is that the piezo regulator is made up of a PI (Proportional-Integrative) controller. Figure B.3 shows the error signal and transmitted intensity when the lock is on or not and these signals remain identical to the ones obtain for the laser frequency stabilization.

In the case shown on the figure B.3 the regulation is not so good since the amplitude of the ‘noise’ should be less than half the amplitude of the slope of the error signal. This mean that the cavity length is stable in average but with length variation too important. One can also remark that when the cavity is looked the level of the transmitted intensity (in red behind the usual transmitted intensity) is almost at the maximum of the usual transmitted intensity, this means that the cavity length almost matches with the top of the resonance (with a good length stabilization this should be exactly on top of the resonance so this is another proof that the stabilization is not optimal).

**B.3 Measurements of the cavity characteristics**

The laser beam from the 793 nm has a power of more than 1 mW which leads to 730 µW at the entrance of the cavity which is a lot for stabilization application. This is actually very useful because the beam does not need to be perfectly well aligned in the cavity to produce an error signal with enough amplitude.

The figure B.4 shows the reflected intensity from the 793 nm laser. The carrier has an amplitude of about 600 mV what will assure a very good error signal. It is barely possible to effectuate any measurements on this signal since it is usually very noisy.
B. Stabilization of the Fabry-Pérot cavity with a 793 nm laser

Fig. B.3: Error signal and transmitted intensity out of lock (in blue) and when the lock is on (in red).

Fig. B.4: Reflected intensity from the 793 nm laser. One can see the time modulation of the sidebands which do not appear as fully reflected, otherwise the shape looks like the theoretical one.

The measurement of the FSR is accomplished with an important scan amplitude to be able to visualize two modes on the same ramp of the piezo. The problem of the time/frequency scale remains the same so the method used is the one explained in section 3.4.2 with a modulation frequency of 20 MHz here. The measurement is shown on figure B.5.

Three measurements have been done on different days to study the stability of the cavity (even if the FSR theoretically depends only on the quality of the mirrors). The errors in these measurements essentially come from the uncertainty on the distance between the two sidebands (about 1% of the distance). Indeed, the amplitude of the scan necessary to see two sideband modes the sideband is too large such that the sidebands are very small and not very sharp. On the contrary the distance between the two maxima of the carriers is very accurate (less than 0.01%) since these ones are very sharp. This leads to:

$$FSR = 485 \pm 9, 508 \pm 5, 515 \pm 4 \text{ MHz}$$

That is in average:

$$FSR = 503 \pm 6 \text{ MHz}$$

which is very close to the theoretical value of 500 MHz.

As explained in the section 2.2 the cavity linewidth $\Delta \nu$ can be measured with the slope of the error signal or with the FWHM (Full Width at Half Maximum) of the transmitted intensity. In any
case a scale between time and frequency will still be required but for this measurement the distance between the two sidebands can be measured very precisely (the error is about 0.1% of the distance) since as shown on the figure B.6 the limits are the zero-crossing of two very steep slopes. So the scale reference will be measured with the error signal. Measurements on the transmitted intensity are also precise since the signal is very sharp (less than 1% of the distance), on the contrary the measurements with the slope of the error signal are not that precise since the top of the signal is usually very noisy (about 5% of the distance). The principle of these measurements is presented on the figure B.6.

Fig. B.5: Principle of the FSR measurements.

Fig. B.6: Transmitted intensity and error signal associated for the linewidth measurement.
Results with the slope of the error signal:

\[ \Delta \nu = 1.83 \pm 0.10, \ 1.89 \pm 0.15 \text{MHz} \]

So an average of \( \Delta \nu = 1.86 \pm 0.13 \text{ MHz} \).

And with the transmitted intensity:

\[ \Delta \nu = 4.63 \pm 0.03, \ 3.30 \pm 0.06, \ 3.18 \pm 0.15, \ 4.73 \pm 0.09, \ 4.32 \pm 0.13, \ 5.35 \pm 0.17 \text{MHz} \]

In average:

\[ \Delta \nu = 4.25 \pm 0.11 \text{MHz} \]

The theoretical value is about 800 kHz, so one can see that the measurements are quite far from that. Moreover measurements based on the slope of the error signal yield a linewidth less than half the one found with the transmitted intensity. No good explanation has been found for this difference so this remains an open question. A coarse measurement done with the reflected intensity gives a value close to the one with the transmitted intensity so the reference value used for the laser linewidth measurement will be the one given by the error signal.

These measurements lead to a Finesse of:

\[ \mathfrak{F} = \frac{FSR}{\Delta \nu} = 270 \pm 22 \]

what is about half of the theoretical value. The gap between the theoretical and the experimental value of the Finesse should not be interpreted as an inaccuracy of the linewidth measurement (even it seems to be the furthest from the theoretical value). The explanation is rather that the quality of the mirrors is not as good as specified and thus the Finesse is not the theoretical one.
C. LOCKING CHAIN

As we have seen in the previous appendix the cavity length is stabilized with the 793 nm laser so this laser needs to be stabilized in frequency. The locking chain ensuring the stabilization is shown in the figure C.1. The principle is the following: the laser beam from a 852 nm laser saturates a Cesium cell which provides an absolute error signal which is constant in time (the error signal is done by Doppler-free spectroscopy on the Cs gas). This error signal is used to stabilized the length of a cavity. The beam of the 852 nm is also sent into the same cavity in order to create an error signal and stabilizes the laser frequency. These 3 elements (852 nm laser, first cavity and Cs cell) compose the reference block for the stability.

Then the beam of the 793 nm is sent into the stabilized cavity in order to get an error signal and to stabilize its frequency.

Finally the now stabilized 793 nm is also sent to the cavity used by the 732nm in order to stabilize the cavity length as explained in the previous parts. The three modules of the lasers (laser box, Toptica PDH and Toptica PID) have been explained in the section 3.1. One important difference between all those lasers and the 732 nm laser is that all other lasers are current modulated which creates sidebands in the frequency domain limiting spectroscopic applications. On the contrary the 732 nm laser is phase modulated only in the locking beam so the output is not modulated and consists of a single frequency.
Fig. C.1: Stabilization chain. On this figure are missing the 866 nm, 854 nm, 850 nm lasers and their cavities. All these lasers are frequency stabilized their own cavity which are stabilized on the 852 nm laser. The solid colored lines are optical paths and dashed lines are electrical signals.
D. POLARIZING BEAM SPLITTER SET-UP

As shown in figure D.1, the output beam from the laser box is more or less linear polarized. The PBS decomposes the incoming beam in the vertical and horizontal directions: about 95% (resp. 5%) of the horizontal (resp. vertical) direction is transmitted while about 5% (resp. 95%) is reflected to the left (on figure D.1). Then the $\lambda/2$-plate allows one to send more or less power to the “loss” direction. The $\lambda/4$-plate and the $\pi$-phase shift induced by the mirror changes the polarization in such way that the light which goes back to the PBS is vertical polarized and then is mostly reflected. The reflection can thus be used.
D. Polarizing Beam Splitter set-up

PBS : Polarizing Beam Splitter

Fig. D.1: PBS set-up.
BIBLIOGRAPHY


