

# Spatial and spatio-temporal wave localization in periodic media

*Invited paper*

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Session: Localized Waves by Hugo E. Hernandez-Figueroa

**Abstract.** We present an analysis of spatial three-dimensional wave localization in one- and two-dimensional photonic crystals (PCs) without defects together with an analytical framework, based on the Wannier function technique, to study spatio-temporal wave localization in more general photonic crystals. We present the results of those analyses with a particular emphasis on hyperbolic wave localization. We show that, in one-dimensional photonic bandgaps, wave diffraction due to localization in the translational-invariant directions is deeply affected by the band structure arising from the periodicity of the photonic crystal, leading to different kinds of wave localization. In particular, for a periodic layered structure we show that close to the bandgap edge, diffraction is enhanced with a transition from a parabolic diffraction curve (Gaussian beam propagation) to hyperbolic or elliptic diffraction curves which allow for localization in the form of stationary X-shaped or sinc-shaped waves respectively. In two-dimensional photonic crystals we study out-of-plane propagation and show the existence of two types of X waves at frequencies corresponding to either a top-point of a band, where the in-plane diffraction turns out to be negative, or at band saddle points. In the former case the X-wave is directed along the invariance direction of the PC, whereas in the latter case it lies in the PC plane and is directed along one of the principal direction of the diffraction tensor. Eventually, in order to study spatio-temporal propagation of light in generic PCs (one- to three- dimensional), we use a Wannier function expansion technique which leads to the description of light propagation through an envelope function. In this framework, different kinds of propagating and stationary spatio-temporal localized waves are found and moreover a class of non-diffractive and non-dispersive wave propagation is presented. Analytical predictions of light behaviour were also checked with numerical computation.