Measurement of electric field by interferometric spectral trace observation

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We present a new methodology that obtains, in an analytical way, the complex electric field of ultrashort pulses. This methodology is based only on Fourier analysis of the frequency components of spectrally resolved interferometric collinear autocorrelations. We present an experimental demonstration of this technique and the results are compared with the conventional second-harmonic generation frequency-resolved optical gating technique. © 2005 Optical Society of America

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The phase of ultrashort pulses is becoming an important tool for research in several fields of knowledge. Phase-dependent interaction between matter has been successfully exploited for applications that range from coherent spectroscopy[1] to nonlinear imaging.[2] These techniques rely on modifying the phase of pulses, through feedback processes, to optimize the desired effect. Knowledge of the exact characteristics (amplitude and phase) of the interacting ultrashort pulses will continue to lead to further interesting insights about the physical phenomenon involved. The problem of how to extract the phase information of an ultrashort pulse when only its spectral intensity can be experimentally measured has therefore resulted in concerted effort. There are now a number of techniques that circumvent this problem. These can be divided into two main categories: time-frequency[3,4] and interferometric[5,6] techniques. Time-frequency techniques generally allow for analysis that immediately detects systematic errors. They, however, rely on the acquisition of many data points as well as an iterative retrieval algorithm[7] to recover the pulse information. In contrast, interferometric techniques[5,6] offer direct phase measurement without the need for retrieval algorithms or the collection of large data sets. As a consequence, pulse characterization can be carried out in real time.[8] Interferometric techniques, however, do not have a stringent error-checking capability; they normally rely on a pulse-specific optical arrangement. In this Letter we describe a new and important general methodology, based on Fourier analysis,[9,10] that allows the phase of an unknown pulse to be analytically obtained. Furthermore, this novel method helps to bring together time-frequency and interferometric techniques while maintaining the robust error-checking capability of the time-frequency approaches and discarding some of their negative attributes. In addition our methodology requires only a simple collinear autocorrelator whose output is spectrally resolved as a function of delay. The experimental setup is shown in Fig. 1. This method is referred to by us as measurement of electric field by interferometric spectral trace observation (MEFISTO).

To start with our analysis, consider a pulse interacting collinearly within a nonlinear crystal after passing through an autocorrelator. The second-harmonic-generated signal is then directed to a spectrograph to obtain an interferometric trace in terms of time delay τ and frequency f. An experimental example of such a trace can be seen in Fig. 2(a). The resulting trace can be mathematically described as

$$I^{\mathrm{SHG}}(f,\tau)=|F_t(\{E(t)\exp(i2\pi f_0 t) + E(t-\tau)\exp[i2\pi f_0(t-\tau)]\}^2)|^2. \qquad (1)$$

Here $E(t)$ is the slowly varying amplitude of the complex electric field centered at frequency f_0. The Fourier transform with respect to variable t is indicated by F_t. The main difference between this and second-harmonic generation (SHG) frequency-resolved optical gating (FROG) is that all the cross terms in Eq. (1) are retained and, as we will show below, with the new information carried on these terms it is possible to analytically obtain $E(t)$.

To do this, we first calculate the Fourier transform of Eq. (1) in the τ axis, $Y^{\mathrm{SHG}}(f,\kappa)=F_\tau[I^{\mathrm{SHG}}(f,\tau)]$. The resulting expression consists of five main spectral components [see Fig. 2(b)] at frequencies $f=0$, $\pm f_0$,

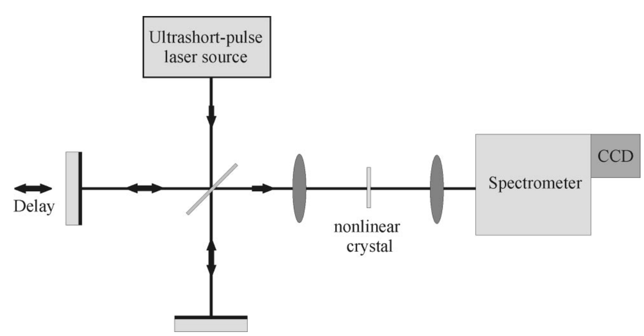

Fig. 1. Schematic of the experimental setup used for MEFISTO.

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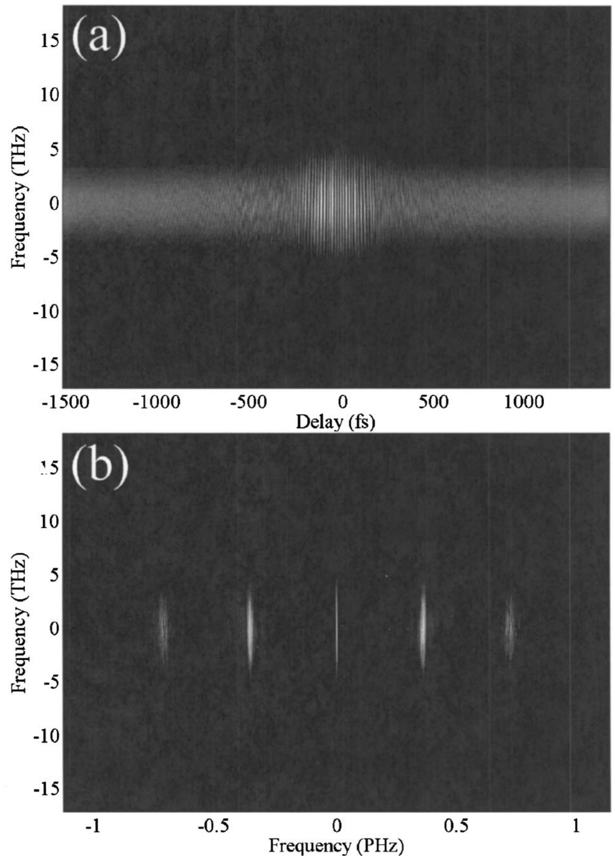

Fig. 2. (a) Frequency-resolved collinear autocorrelation. (b) Same trace in the Fourier domain. (For clarity, the intensity scale is not linear.)

and $\pm 2f_0$. Since the interferometric trace [Fig. 2(a)] is symmetric and real, the negative spectral components are real and equal to the positive ones [see Fig. 2(b)]. Therefore, to analyze the information enclosed in the transformed trace, we only need to focus on the positive frequency components. Each of these terms carries information about the pulse phase and intensity, and their use depends on the particular experimental conditions.[10]

Here we focus on and analyze the most interesting case of the spectral components of $Y^{\mathrm{SHG}}(f,\kappa)$ near $\kappa \approx f_0$. We can rewrite this spectral component as

$$Y^{\mathrm{SHG}}_{\kappa\approx f_0}(f,\kappa) = 2E_{\mathrm{SHG}}(f)E^*(f+f_0-\kappa)E^*(\kappa-f_0) + \text{c.c.}, \tag{2}$$

where $E_{\mathrm{SHG}}(f)=\int_{-\infty}^{\infty}\mathrm{d}f' E(f')E(f-f')$ and $E(f)=F_t\{E(t)\}$. With Eq. (2) it is possible to determine $E(t)$ in an analytical way. To show this, we write the spectral component of the complex electric field in polar form, $E(f)=U(f)\exp[i\phi(f)]$. Then Eq. (2) can be written as

$$Y^{\mathrm{SHG}}_{\kappa\approx f_0}(f,\kappa) = 4U_{\mathrm{SHG}}(f)U(f+f_0-\kappa)U(\kappa-f_0)\cos[\phi_{\mathrm{SHG}}(f) - \phi(f+f_0-\kappa) - \phi(\kappa-f_0)]. \tag{3}$$

Under typical laboratory conditions the spectral amplitude of the fundamental pulses, $U(f)$, and of the corresponding second harmonic, $U_{\mathrm{SHG}}(f)$, are known. Therefore the only unknowns in Eq. (3) are the spectral phases of the fundamental and second-harmonic pulses, $\phi(f)$ and $\phi_{\mathrm{SHG}}(f)$. When the phases are known, the pulses are completely characterized. This can be successfully achieved by taking two different slices in the transformed space of the interferometric trace, e.g., at $\kappa=f_0$ and $\kappa=f_0-\Delta f$. Then from Eq. (3) we obtain

$$\phi_{\mathrm{SHG}}(f) - \phi(f) - \phi(0) = \pm\cos^{-1}[\Omega(f,\kappa=f_0)], \tag{4a}$$

$$\phi_{\mathrm{SHG}}(f) - \phi(f+\Delta f) - \phi(-\Delta f) = \pm\cos^{-1}[\Omega(f,\kappa=f_0-\Delta f)], \tag{4b}$$

where we have defined $\Omega(f,\kappa)=Y^{\mathrm{SHG}}(f,\kappa)/4U_{\mathrm{SHG}}(f)U(f+f_0-\kappa)U(\kappa-f_0)$. Note that all the functions in parameter $\Omega(f,\kappa)$ can be experimentally obtained. Then, by subtracting Eqs. (4a) and (4b), we get

$$\Delta\phi(f) = \phi(f+\Delta f) - \phi(f) = \pm\cos^{-1}[\Omega(f,\kappa=f_0)] \mp \cos^{-1}[\Omega(f,\kappa=f_0-\Delta f)] + \phi(0) - \phi(-\Delta f). \tag{5}$$

This equation is the final result of the MEFISTO method and allows the determination of the phase of $E(f)$ by taking an arbitrary origin $\phi(0)$ and varying f. However, to use Eq. (5), there are some aspects that are worth noting. First, the term $\phi(0)-\phi(-\Delta f)$ in Eq. (5) is a constant that can be decided arbitrarily. This term adds a linear phase shift that is equivalent to determining the electric field origin in time. Second, in the theoretical development a $\cos^{-1}(\Omega)$ function is used, and as a consequence the sign of the phase shift is not determined. This results in two possible solutions, $E(f)$ and $E^*(f)$, which is the characteristic ambiguity that also appears in FROG measurements and cannot be resolved using schemes based on quadratic nonlinearities. Additionally, for simplicity of the analysis we implicitly consider that the sampling step in the f and κ axis coincides ($\Delta\kappa=\Delta f$).

To show the validity of the MEFISTO methodology, we experimentally obtain the spectral phase of pulses originating from a Kerr-lens mode-locked Ti:sapphire laser. The laser has a central wavelength of 800 nm and a repetition rate of 76 MHz. The laser beam is focused into a type I β-barium borate crystal through a Michelson interferometer (autocorrelator). The SHG signal is sent to a spectrometer and detected by a CCD linear array. The obtained frequency-resolved interferometric autocorrelation trace is the one shown in Fig. 2(a). To resolve the interferometric fringes we choose a delay step of $\Delta\tau=0.44$ fs, which follows Nyquist criteria. We choose a time-delay span of $\tau_{\mathrm{span}}=3$ ps that results in a frequency step of $\Delta f=0.33\ \mathrm{ps}^{-1}$ and a spectral resolution of $\Delta\lambda=0.17$ nm. We then analytically extract the spectral phase of our pulse using Eq. (5). To demonstrate the effectiveness of MEFISTO, we use the same experimental data to analytically characterize the pulse and then compare them with a standard SHG FROG retrieval. The SHG FROG trace is obtained with the collinear

FROG technique.[10] Marginal analysis is also carried out to ensure that errors are not present within the trace. It should be emphasized that MEFISTO can use the same stringent marginal checks. The results are outlined in Fig. 3, in which the spectral intensity and phase of the pulse obtained from both techniques are compared. We can see that, although the methodologies used to obtain the pulse characteristics are completely different, the intensity and phase are similar. As a further evaluation, the calculated interferometric autocorrelations from both methods are compared with the experimental one, obtained by integrating the interferometric trace in the frequency axis. These results are shown in Fig. 4 and show good agreement.

In conclusion, in this work we have outlined a new procedure based on a simple collinear autocorrelator that allows the complex amplitude of ultrashort pulses to be analytically deduced. The technique relies on Fourier analysis after obtaining a spectrally resolved interferometric autocorrelation trace. The MEFISTO methodology has the crucial advantage over SHG FROG that it allows the simple extraction of pulse information without the need for an iterative retrieval algorithm. Furthermore, it still maintains the powerful error-checking capabilities that are associated with time-frequency techniques. We have experimentally demonstrated the effectiveness of the new procedure by comparing results with the more traditional characterization technique of SHG FROG using an identical optical arrangement. This setup is extremely flexible and simple, allowing a large number of different applications to be carried out.[11]

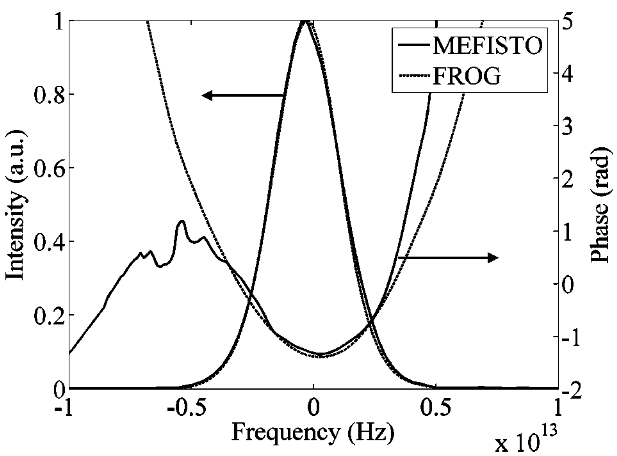

Fig. 3. Spectra and phases of the pulse obtained with MEFISTO (solid curves) and a standard SHG FROG procedure (dashed curves).

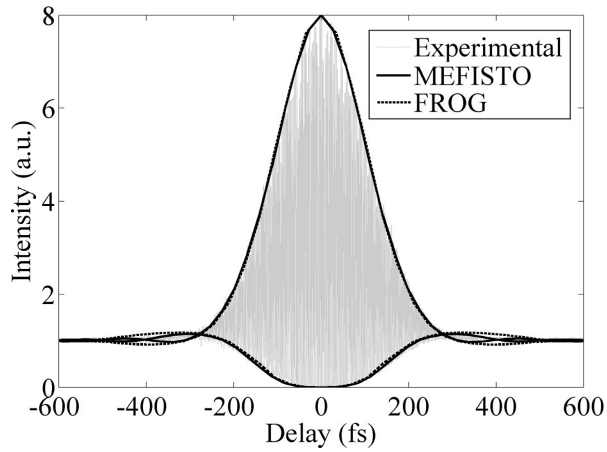

Fig. 4. Numerical interferometric autocorrelations obtained from MEFISTO (solid curve) and the SHG FROG technique (dashed curve) compared with experimental measurements (in light gray).

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