Generation of surface soliton arrays

Yaroslav V. Kartashov, Victor A. Vysloukh, Dumitru Mihalache, and Lluis Torner
ICFO-Institut de Ciencies Fotoniques, and Universitat Politecnica de Catalunya, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain

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We discover that, at the edge of an optical lattice imprinted in a saturable nonlinear medium, one-dimensional surface solitons can exist only within a band of light intensities and that they cease to exist when the lattice depth exceeds an upper threshold. We also reveal the generation of arrays of two-dimensional surface solitons mediated by the transverse modulational instability of one-dimensional solitons, a process that is found to exhibit specific features associated to properties of the optical lattice. © 2006 Optical Society of America

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The presence of interfaces between linear and nonlinear materials strongly affects the propagation of intense laser beams.1–3 Such interfaces can support solitons localized at the very interface. Until recently, progress in the experimental observation of surface solitons has been limited by the huge powers required for their excitation at the interfaces of natural materials. Thus surface solitons were observed only at interfaces of linear and photorefractive media4 and at the interface with linear layered media.5 Recently, the concept of surface solitons in semi-infinite cubic waveguide arrays was proposed,6–8 which resulted in the prediction and observation of new types of surface solitons.9,10 All such solitons observed to date are one-dimensional (1D). However, the technique of optical induction11–14 allows the creation of truly two-dimensional (2D) periodic interfaces.

In this Letter we study the properties of solitons supported by the interface of an optical lattice with saturable nonlinearity and a linear medium. We find that in such a setting 1D surface solitons exist only inside a band of light intensities and that they cease to exist when the lattice depth exceeds a critical value, the latter being a new effect arising due to the competition between nonlinearity saturation and surface effects. 1D solitons are prone to transverse modulational instabilities (TMIs), previously studied for interfaces of uniform media,15–17 which may generate arrays of bulk, or 2D surface solitons. Here we explore such a process for soliton arrays generated at the edge of an optical lattice.

We consider the propagation of light along the ξ axis of a biased photorefractive crystal. A pair of interfering plane waves induces a periodic refractive index modulation along the η axis, while along the second transverse ω axis the refractive index is uniform. An intense green-light background illumination may be used to produce a region with high photocconductivity where the red-light imprinted lattice is erased, thus creating an interface between a periodic nonlinear medium and a uniform linear medium. The transition region can be made sharp by blocking the background wave in the space η>0, so that diffraction is negligible on the scale of tenths of soliton widths. We assume that the soliton beam and interface-creating waves are orthogonally polarized to each other. In this case the complex amplitude of the light field evolves according to:

\[
\frac{i \partial q}{\partial \xi} = -\frac{1}{2} \left( \frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \omega^2} \right) - Eq \left( \frac{|q|^2 + pR(\eta)}{1 + |q|^2 + pR(\eta)} \right) \quad \text{at } \eta \geq 0,
\]

\[
\frac{i \partial q}{\partial \xi} = -\frac{1}{2} \left( \frac{\partial^2 q}{\partial \eta^2} + \frac{\partial^2 q}{\partial \omega^2} \right) \quad \text{at } \eta < 0. \tag{1}
\]

Here the longitudinal ξ and the transverse η, ω coordinates are scaled to the diffraction length and to the beam width, respectively; the parameter E describes the static biasing field applied to the crystal; p is the lattice depth; and the function \( R(\eta) = \sin^2(\Omega_0 \eta) \) stands for the profile of a lattice with frequency \( \Omega_0 \). We set \( \Omega_0 = 2 \). Our parameters correspond to beam widths \( \sim 10 \mu m \) at the wavelength 0.63 \( \mu m \), launched into a strontium barium niobate crystal with an electro-optical coefficient \( r_{eff} = 1.8 \times 10^{-10} \) m/V, biased with a static electric field \( \sim 10^4 \) V/m. Then \( \xi = 1 \) corresponds to 2 mm, and \( q = 1 \) corresponds to peak intensities \( \sim 10 \) mW/cm². In strontium barium niobate the nonlinear coefficient for extraordinary-polarized soliton beams may exceed that of ordinary-polarized lattice beams by a factor of 20, ensuring that the lattice experiences no back action from the soliton.

First we address the properties of 1D solitons supported by such interfaces. We search for them in the form \( q(\eta, \xi, \omega) = w(\eta) \exp(i b \xi) \), where \( w(\eta) \) is a real function and b is the propagation constant, and assume that the light field does not depend on \( \omega \). The quantity \( U = \int_{-\infty}^{\infty} |q|^2 \, d\eta \) gives the energy flow carried by the solitons. Their profiles can be found from Eq. (1) numerically with a relaxation method. To analyze soliton stability we search for perturbed solutions of Eq. (1) in the form \( q = (u + u_p + iv_p) \exp(i b \xi) \), where \( u_p = u(\eta) \exp(i \delta \xi) \cos(\Omega \xi) \) and \( v_p = v(\eta) \exp(i \delta \xi) \cos(\Omega \xi) \) are the real and the imaginary parts of perturbations that can grow with rate \( \delta \), while \( \Omega \) describes the transverse modulation frequency. Substitution of these expressions into Eq. (1) and linearization around stationary solution \( w \) yields the eigenvalue problem.

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The central issue addressed in this Letter is the evolution of the 1D solitons when higher-dimensional dynamics along the transverse coordinate $\zeta$ is included. In this case, the development of TMI of surface-wave stripes along $\zeta$ may result in the formation of 2D soliton arrays. To elucidate the specific features of TMI at interfaces with lattice, we calculated the growth rates for odd and even solitons at modulation frequencies $\Omega>0$ (Fig. 3). All instabilities that were encountered are of exponential type. The growth rate is nonzero only within a finite frequency band: $\delta$ vanishes for odd solitons and remains nonzero for even solitons when $\Omega \rightarrow 0$.

Direct integration of Eq. (1) enables us to determine which structures emerge in the developed stage of TMI. We solved Eq. (1) with the input conditions $q|_{\zeta=0}=w+(u+iv)\cos(\Omega\zeta)$, where $u,v\ll w$ are obtained from Eq. (2). Thus input field distributions are bright near-surface stripes localized along the $\eta$ axis and
one filament with scale 2

tion frequencies, when energy concentrated within
typical regimes were encountered. For small modula-
tent with limited existence domain in
beams with too high or too low energy flows, consis-
tlinear medium. More important, the interface repels
accompanied by strong radiation emission into the
feature of TMI at the interface is that decay may be
surface or form sets of 2D surface waves. The specific
fragments [Fig. 4(a)] that either move away from the
2D solitons, the 1D surface waves break into many

shallow modulation of surface waves occurs without
wave breakup [Fig. 4(c)]. Simulations show that ar-
rays of well-localized 2D surface waves emerging due
to TMI of 1D waves are robust and may propagate
undistorted over huge distances [e.g., the array de-
picted in Fig. 4(b) keeps its structure for more than
10^4 propagation units, exceeding any experimentally
feasible crystal lengths by several orders of magni-
tude]. By tuning the energy flow of input quasi-1D
waves and the frequency of transverse modulation
one may generate arrays with various separations
between neighboring spots.

We thus conclude by stressing that we introduced
the properties of solitons attached at the surface of a
lattice imprinted in media with saturable nonlinearity.
The existence conditions of such solitons exhibit
new threshold effects versus energy flow and lattice
depth. We showed how TMIs of 1D solitons lead to
the generation of robust 2D surface solitons arrays.

V. A. Vysloukh is also with Universidad de las
Americas, Puebla, Mexico. D. Mihalache is with In-
stitute of Atomic Physics, Bucharest, Romania.
Y. V. Kartashov’s e-mail address is
yaroslav.kartashov@icfo.es.

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