Tailoring the spectral coherence of heralded single photons

Víctor Torres-Company,[1,2] Alejandra Valencia,[1] and Juan P. Torres[1,3,*]

[1]*ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park 08860 Castelldefels (Barcelona), Spain*
[2]*Departament de Física, Universitat Jaume I, ES-12071 Castelló, Spain*
[3]*Department of Signal Theory and Communications, Universitat Politecnica de Catalunya, 08034 Barcelona, Spain*
Corresponding author: juan.perez@icfo.es

Received December 23, 2008; accepted February 25, 2009;
posted March 16, 2009 (Doc. ID 105736); published April 7, 2009

Currently, different techniques are known for synthesizing in a tunable fashion the frequency correlations of paired photons generated in spontaneous parametric downconversion (signal-idler correlations). We show that these same techniques can also be used to tune the first-order coherence properties of the signal or idler photons (signal–signal or idler–idler frequency correlations). In consequence, partially coherent radiation can be synthesized through adequate biphoton engineering. Interestingly, when the biphoton is designed to be in a separable state, the signal photons behave as a broadband first-order coherent light source. © 2009 Optical Society of America

OCIS codes: 030.1640, 190.4410, 270.0270.

Spontanesous parametric downconversion (SPDC) is an optical process, where a nonlinear crystal is pumped by an intense beam mediating the generation of two photons (referred as signal and idler) that satisfy energy and momentum conservation. The two-photon probability amplitude (or biphoton) that describes the quantum state of this light can show quantum correlations in frequency, momentum, and polarization degrees of freedom. This makes SPDC a ubiquitous source of photon pairs in quantum information processing tasks [1]. Additionally, the simultaneous creation of the pair makes this light useful for several applications in, e.g., coherent anti-Stokes Raman scattering [2] or as a tool for the generation of heralded single photons [3].

In recent years, applications requiring a specific bandwidth and different types of two-photon frequency correlations (anticorrelation, correlation, and even uncorrelation) have triggered the development of methods to control the frequency properties of the biphoton in a tunable way. We mention, e.g., the use of appropriate materials and wavelengths [4,5], introducing angular dispersion into the pump and downconverted beams [6], inserting phase [7] and/or amplitude [8] spectral filters in signal and idler beams, using chirped quasi-phase-matched materials [9], the so-called spatial-to-spectral mapping technique [10], or the combination of angular dispersion and noncollinear geometries [11].

For the use of heralded single photons based on SPDC sources, it is important to assess the characteristics of the intrabeam (signal–signal or idler–idler) correlations. In the cw pump configuration in which the biphoton corresponds to the frequency anticorrelated case, it is widely accepted that the single-mode intrabeam correlations correspond to that of a stationary light beam obeying Gaussian statistics [12,13]. Joobeur *et al.* [14] analyzed the case for a broadband pump pulse, but owing to the time average on the first-order correlations, only partial information on the coherence properties can be obtained. Ou *et al.* [15] showed that the intrabeam first-order coherence properties are connected to the two-photon probability amplitude of the signal-idler pair.

Based on this connection, in this Letter we illustrate how the intrabeam first-order coherence can be tuned through appropriate engineering of the frequency entanglement in SPDC. We show that, in general, partial frequency entanglement between signal and idler leads to spectrally partially coherent radiation for the intrabeam photons. As a limit case, full anticorrelation or correlation in signal-idler frequencies leads to a spectrally incoherent (stationary) signal light beam. We also find that signal (or idler) photons show first-order coherence when the biphoton is designed to be in a separable state. We point out that this was thought to be possible only through narrowband filtering [15]. However, thanks to the new developments in spectral biphoton engineering, it is now possible to achieve this without the need of filtering. Then, an unfiltered separable biphoton constitutes a source of heralded single photons that behave as first-order coherent pulses.

Consider an SPDC source where one of the photons is detected with an ideal broadband detector, revealing the presence of the other photon. Our aim is to calculate the signal first-order frequency correlation function, $W(\omega_1,\omega_2)$. According to Glauber's theory,

$$W(\omega_1,\omega_2)=\mathrm{Tr}[\rho_s a_s^\dagger(\omega_1)a_s(\omega_2)], \tag{1}$$

where $a_s(\omega)$ and $a_s^\dagger(\omega)$ are the annihilation and creation operators of a signal photon at frequency ω and ρ_s is the signal's density matrix obtained by tracing out the idler in the two-photon state; $\rho_s=\mathrm{Tr}_i[|\Psi\rangle\langle\Psi|]$. Here $|\Psi\rangle$ is the SPDC quantum state, which using a first-order perturbation theory and considering a classical coherent pump pulse can be written as $|\Psi\rangle=\int\int d\omega_s d\omega_i \Phi(\omega_s,\omega_i)a_s^\dagger(\omega_s)a_i^\dagger(\omega_i)|0,0\rangle$, where $|0,0\rangle$ denotes the vacuum state and the subscript i refers to the idler photon. The amplitude probability distribution or biphoton, $\Phi(\omega_s,\omega_i)$, depends on the crystal configuration and the pump parameters. When properly normalized, $|\Phi(\omega_s,\omega_i)|^2$ gives the joint probability of having one signal photon at frequency ω_s and one idler at ω_i.

Using $|\Psi\rangle$, ρ_s becomes

$$\rho_s = \int\int\int \mathrm{d}\omega \mathrm{d}\omega_s \mathrm{d}\omega_s' \Phi(\omega_s,\omega)\Phi^*(\omega_s',\omega)a_s^\dagger(\omega_s)|0\rangle \times\langle 0|a_s(\omega_s'). \quad (2)$$

Placing this expression in Eq. (1) and using the bosonic conmutation rules $[a_s(\omega_s),a_s^\dagger(\omega_s')]=\delta(\omega_s-\omega_s')$, we get

$$W(\omega_1,\omega_2) = \int \mathrm{d}\omega \Phi^*(\omega_1,\omega)\Phi(\omega_2,\omega), \quad (3)$$

in full agreement with [15]. Equation (3) shows the interplay between interbeam and intrabeam correlations. It is interesting to note that $\int\int \mathrm{d}\omega_1 \mathrm{d}\omega_2 |W(\omega_1,\omega_2)|^2$ is the meaningful quantity that determines the visibility in different multiphoton interference schemes [15]. Here, we stress that $\int \mathrm{d}\omega \Phi^*(\omega_1,\omega)\Phi(\omega_2,\omega)$ has a meaningful physical interpretation in terms of first-order spectral correlations. Furthermore, $W(\omega_1,\omega_2)$ can be demonstrated to be nonnegative definite [16]. This characteristic is a necessary requirement for a correlation function to be genuinely defined from a classical point of view [13].

According to Eq. (3), the intrabeam correlations are linked to the entanglement properties enclosed in the biphoton. Since the biphoton can be synthesized with the above mentioned techniques, we claim that the first-order coherence properties of the signal photons can also be tailored in a user-defined fashion. Strictly speaking, one only gets statistically stationary light when $W(\omega_1,\omega_2)\propto\delta(\omega_2-\omega_1)$, which implies that the radiation is spectrally incoherent. This may be achieved when the biphoton is either fully anticorrelated, i.e., $\Phi(\omega_s,\omega_i)\propto\delta(\omega_s+\omega_i)$, which is the case for cw pumping, or fully correlated, $\Phi(\omega_s,\omega_i)\propto\delta(\omega_s-\omega_i)$.

On the other hand, when the biphoton is engineered to be uncorrelated, or separable, $\Phi(\omega_s,\omega_i)\propto f(\omega_s)g(\omega_i)$ (where f and g are independent functions describing the spectral properties of signal and idler, respectively), from Eq. (3) one obtains that the first-order correlation function becomes separable also, i.e., $W(\omega_1,\omega_2)\propto f^*(\omega_1)f(\omega_2)$. This indicates that the signal photons behave as first-order coherent pulses.

To further illustrate the above mentioned characteristics, let us particularize to the case of an engineered biphoton depicted by a Gaussian function, $\Phi(\omega_s,\omega_i)=\exp[-(\omega_s-\omega_i)^2/B_-^2]\exp[-(\omega_s+\omega_i)^2/B_+^2]$, where B_- and B_+ are real constants related to the width along the antidiagonal (slope at $-45°$) and diagonal (slope at $+45°$) in the (ω_s,ω_i) plane. These parameters determine the amount of frequency entanglement. Figures 1(a)–1(c) show the different biphoton spectral correlations that could be achieved by properly engineering the biphoton amplitude [4–6,10,11]. The corresponding intrabeam first-order correlation function, $W(\omega_1,\omega_2)$, is shown in Figs. 1(d)–1(f). As can be seen, both anti- and correlated cases for the biphoton lead to an inclination of W in the (ω_1,ω_2) plane, indicating that we have spectrally partially coherent radiation [17]. On the other hand, a fully unentangled state leads to a spectrally fully coherent pulse, which is revealed by the circular shape of W in Fig. 1(e). Finally, although not illustrated in the plots, which represent the nonstationary case, a perfectly anticorrelated (or correlated) biphoton would show an infinitely narrow line along the $\omega_1=\omega_2$ direction, corresponding to statistically stationary light.

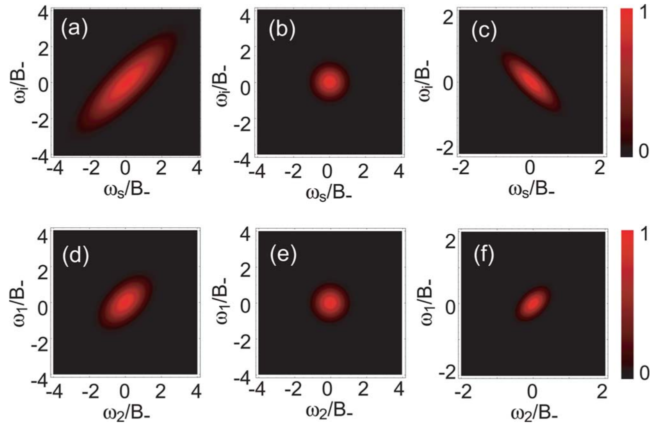

Fig. 1. (Color online) Biphoton engineering and corresponding first-order coherence for the signal photons. (a)–(c) Plot $\Phi(\omega_s,\omega_i)$ for (a) highly correlated, $B_-=0.3B_+$; (b) uncorrelated, $B_-=B_+$; and (c) highly anticorrelated, $B_+=3B_-$ paired photons. (d)–(f) Corresponding intrabeam cross-spectral density functions $W(\omega_1,\omega_2)$. Note the change of scale in (c) and (f).

Alternatively, first-order correlations can be described in the time domain. The mutual coherence function of the signal photons can be obtained via the generalized Wiener–Khintchine theorem, $\Gamma(t_1,t_2)=\int\int \mathrm{d}\omega_1 \mathrm{d}\omega_2 W(\omega_1,\omega_2)\exp[-i(\omega_1 t_1-\omega_2 t_2)]$. The temporal complex degree of coherence is $\gamma(t_1,t_2)=\Gamma(t_1,t_2)\times[I(t_1)I(t_2)]^{-1/2}$ with $I(t)=\Gamma(t,t)$. For the previous example of the Gaussian biphoton we get $\gamma(t_1,t_2)=\exp[-(t_2-t_1)^2/(2t_c^2)]$, where t_c is the coherence time whose value depends on the selected choice of the entanglement parameters, i.e.,

$$t_c^2 = 16\frac{B_+^2+B_-^2}{(B_+^2-B_-^2)^2}. \quad (4)$$

It can be noticed that a high-frequency anticorrelated $(B_-\gg B_+)$ or correlated biphoton $(B_-\ll B_+)$ leads to temporally incoherent radiation $(t_c\to 0)$, while the uncorrelated $(B_-=B_+)$ leads to a first-order temporally coherent pulse $(t_c\to\infty)$.

Finally, we propose an experiment where the coherence properties of the signal photon can clearly manifest. The setup is presented in Fig. 2. We consider a source of properly engineered entangled two-photon light. The signal and the idler photons are separated, and the signal is modulated by an external modulator providing a temporal Gaussian profile $m(t)=\exp[-t^2/(2T^2)]$, with T being a measurement of

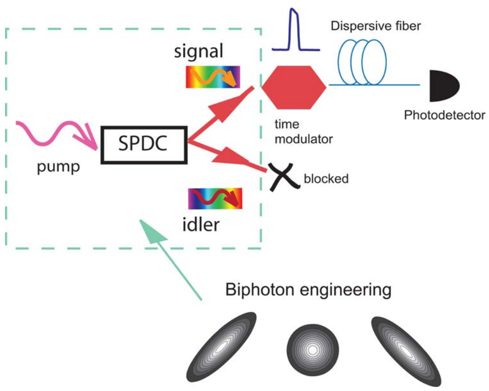

Fig. 2. (Color online) Scheme proposed to measure the probability density distribution of the signal photons.

the gate's temporal duration. Then, the modulated photons are launched into a linear dispersive medium, such as an optical fiber, which can be characterized by a phase-only complex spectral amplitude $H(\omega)=\exp[i\Phi_2\omega^2/2]$, with Φ_2 denoting the group-delay-dispersion coefficient. The probability of detecting a signal photon at a time instant t_s writes $P(t_s)=\int|\Phi'(t_s,t_i)|^2 dt_i$, where $\Phi'(t_s,t_i)$ is the propagated biphoton function. The rms width of the function $P(t_s)$ becomes

$$\sigma^2 = \sigma_0^2\left[1 + \frac{\Phi_2^2}{\tau^4}(1 + \tau^2\sigma_\omega^2)\right], \tag{5}$$

where $\tau=\sqrt{2}\sigma_0$ and σ_0 is the rms width of the probability density distribution with no fiber; $\sigma_0^2=T^2/2[1+T^2(B_-^2B_+^2)/(B_-^2+B_+^2)]^{-1}$; and $\sigma_\omega=t_c^{-1}$ is the equivalent spectral width, which depends on the entanglement characteristics. When the biphoton is highly anticorrelated or correlated, $t_c\to 4B_-^{-1}$ and $t_c\to 4B_+^{-1}$, respectively, in both cases σ obeys the evolution equation of a gate modulating a stationary broadband source [18]. Alternatively, if the biphoton is uncorrelated, we have $t_c\to\infty$, thus the rms of the probability density function matches the evolution equation of a first-order coherent pulse of temporal width $\sigma_0=T/\sqrt{2}[1+T^2B_-^2/2]^{-1/2}$.

Notice that, until now, we have lied in the two-photon regime. However, making use of [19], $|\Psi\rangle$ for the multiple-pair case becomes

$$|\Psi\rangle = \frac{1}{\cosh \eta L}\sum_{n=0}^{\infty} \tanh^n \eta L \frac{(b_s^\dagger)^n}{\sqrt{n!}}\frac{(b_i^\dagger)^n}{\sqrt{n!}}|0,0\rangle \tag{6}$$

when the function $\Phi(\omega_s,\omega_i)$ is designed to be separable. Here L is the crystal length and η denotes the strength of the interaction (nonlinear coefficient and pump beam amplitude), $b_s^\dagger=\int d\omega_s f(\omega_s)a_s^\dagger(\omega_s)$ and $b_i=\int d\omega_i g(\omega_i)a_i^\dagger(\omega_i)$. Signal (idler) photons derived from Eq. (6) are broadband and do show first-order coherence [20]. This implies that the separability of $\Phi(\omega_s,\omega_i)$ ensures first-order coherence even in the multiphoton regime, although the purity still degrades when increasing the photon number.

In summary, we have shown how to tune the first-order coherence properties of signal photons through the recent techniques of biphoton engineering in SPDC. The work presented here opens the door to achieve partially coherent single-photon radiation on demand. This may trigger applications in quantum control, where the synthesis of the coherence properties of light pulses is more important than the individual electric field realizations [21].

This work was supported by the European Commission (Qubit Applications, contract 015848) and by the Government of Spain (GS) (Consolider Ingenio CSD2006-00019, CSD2007-00013, FIS2007-60179, and FIS2007-62217). V. Torres thanks GS for a Formación Profesorado Universitario fellowship.

References

1. Y. H. Shih, Rep. Prog. Phys. **66**, 1009 (2003).
2. C. L. Evans and X. S. Xie, Annu. Rev. Anal. Chem. **1**, 883 (2008).
3. I. A. Walmsley and M. G. Raymer, Science **307**, 1733 (2005).
4. O. Kuzucu, M. Fiorentino, M. A. Albota, F. N. C. Wong, and F. X. Kartner, Phys. Rev. Lett. **94**, 083601 (2005).
5. P. J. Mosley, J. S. Lundeen, B. J. Smith, P. Wasylczyk, A. B. U'Ren, C. Silberhorn, and I. A. Walmsley, Phys. Rev. Lett. **100**, 133601 (2008).
6. M. Hendrych, M. Micuda, and J. P. Torres, Opt. Lett. **32**, 2339 (2007).
7. A. Pe'er, B. Dayan, A. A. Friesem, and Y. Silberberg, Phys. Rev. Lett. **94**, 073601 (2005).
8. F. Zäh, M. Halder, and T. Feurer, Opt. Express **16**, 16452 (2008).
9. M. B. Nasr, S. Carrasco, B. E. A. Saleh, A. V. Sergienko, M. C. Teich, J. P. Torres, L. Torner, D. S. Hum, and M. M. Fejer, Phys. Rev. Lett. **100**, 183601 (2008).
10. A. Valencia, A. Cere, X. J. Shi, G. Molina-Terriza, and J. P. Torres, Phys. Rev. Lett. **99**, 243601 (2007).
11. X. Shi, A. Valencia, M. Hendrych, and J. P. Torres, Opt. Lett. **33**, 875 (2008).
12. B. R. Mollow and R. J. Glauber, Phys. Rev. **160**, 1076 (1967).
13. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U. Press, 1995).
14. A. Joobeur, B. E. A. Saleh, and M. C. Teich, Phys. Rev. A **50**, 3349 (1994).
15. Z. Y. Ou, J. K. Rhee, and L. J. Wang, Phys. Rev. A **60**, 593 (1999).
16. F. Gori and M. Santarsiero, Opt. Lett. **32**, 3531 (2007).
17. P. Paakonen, J. Turunen, P. Vahimaa, A. T. Friberg, and F. Wyrowski, Opt. Commun. **204**, 53 (2002).
18. D. Marcuse, Appl. Opt. **19**, 1653 (1980).
19. P. Kok and S. L. Braunstein, Phys. Rev. A **61**, 042304 (2000).
20. U. M. Titulaer and R. J. Glauber, Phys. Rev. **145**, 1041 (1966).
21. V. Torres-Company, G. Mínguez-Vega, J. Lancis, and A. T. Friberg, Opt. Lett. **32**, 1608 (2007).