Intensity-dependent dispersion under conditions of electromagnetically induced transparency in coherently prepared multistate atoms

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Abstract

Interest in lossless nonlinearities has focussed on the dispersive properties of Λ systems under conditions of electromagnetically induced transparency (EIT). We generalize the Λ system by introducing further degenerate states to realize a "chain Λ" atom where multiple coupling of the probe field significantly enhances the intensity-dependent dispersion without compromising the EIT condition.

1. Introduction

There is increasing interest in studying non-linear optical effects that turn on at the single or few photon level – Quantum Non-Linear Optics. Electromagnetically Induced Transparency (EIT) has been consistently discussed as a method of providing the appropriate conditions. Most of the focus of these investigations has been on atoms in the three level Λ configuration. Although dramatic demonstrations of non-linear effects have been shown (for example slow, stopped and superluminal light) (Matsko *et al* 2001), these effects have been cross modulation experiments, where a strong field affects a weak field. It is difficult to achieve self or weak field-weak field modulation and so attention has shifted towards multi-level atoms for such properties. The N configuration has been studied for a variety of purposes, and has been extended to a 'chain N' configuration for enhanced non-linearity (Zubairy *et al.* 2002). Unfortunately such systems all suffer from the same dilemma, namely that resonance is required for large non-linearities, but simultaneously, this resonance increases dephasing because an excited state is always mixed into the superposition. Here we present an alternative, where the non-linearity is achieved in a system where only ground states contribute to the superposition. We term this the Chain Λ configuration and we describe some of its properties and practical schemes for realization. This work has appeared in publication (Greentree *et al.* 2003)

2. System

The Chain Λ system is a multi-state atom. The lowest order chain Λ is the standard Λ configuration [Fig. 1(a)] with ground states $|g_1\rangle$, $|g_2\rangle$ and excited state $|e_1\rangle$. The next Chain Λ is the M configuration, formed by adding states $|g_3\rangle$ and $|e_2\rangle$ as in Fig. 1(b). One can then generalize this to any arbitrary order of system. In all cases there are selection rules such that the probe excites transitions of the form $|g_n\rangle$-$|e_n\rangle$ and the coupling field excites transitions of the form $|g_{n+1}\rangle$-$|e_n\rangle$.

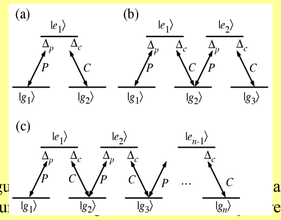

Fig. 1: Energy-level configurations for chain Λ atoms. (a) is the usual Λ system, (b) is the 5-state chain Λ or M system, (c) shows the generalization to a higher number of states.

This configu atomic media, for example an alkali vapour in Doppler-free configu ealize the M system in an $F = 2$ to $F' = 2$ transition. The coupling field is σ^- polarized and turned on before the σ^+ polarized probe. Optical pumping then selects the desired M system over the undesired W system. This scheme generalizes in an obvious fashion. This method is not significantly more complex than the standard schemes used for the observation of EIT in Λ systems.

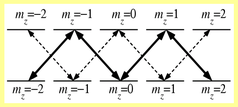

Fig. 2: A possible way to realize the M system using an $F = 2$ to $F' = 2$ transition. The coupling field is σ^- polarized and turned on before the σ^+ polarized probe. Optical pumping then selects the desired M system over the undesired W system.

3. Hamiltonian

The Hamiltonian for the Chain Λ is surprisingly easy to investigate and has a tridiagonal form. In the rotating wave approximation, we can write this:

$$\frac{\mathcal{H}}{\hbar} = \sum_{j=1}^{n} (j-1)(\Delta_p - \Delta_c)|g_j\rangle\langle g_j| + \sum_{j=1}^{n-1} [j\Delta_p - (j-1)\Delta_c]|e_j\rangle\langle e_j| + \sum_{j=1}^{n-1} P_j(|g_j\rangle\langle e_j| + |e_j\rangle\langle g_j|) + \sum_{j=1}^{n-1} C_j(|g_{j+1}\rangle\langle e_j| + |e_j\rangle\langle g_{j+1}|).$$

where Δ_p (Δ_c) is the probe (coupling) detuning, and P_j (C_j) is the probe (coupling) Rabi frequency on the j^{th} transition. We express this Hamiltonian in matrix form and proceed to investigate the eigenstates in the vicinity of resonance, i.e. $\Delta_p = \Delta_c << P_i, C_j$.

$$\frac{\mathcal{H}}{\hbar} = \begin{bmatrix} 0 & P_1 & 0 & 0 & & & \\ P_1 & \Delta_p & C_1 & 0 & & & \\ 0 & C_1 & \Delta_{cp} & P_2 & & & \\ 0 & 0 & P_2 & \ddots & \ddots & & \\ & & & \ddots & (n-1)\Delta_{cp} & P_n & 0 \\ & & & & P_n & n\Delta_{cp}+\Delta_c & C_n \\ & & & & 0 & C_n & n\Delta_{cp} \end{bmatrix},$$

4. Eigenspectra

In Figs. 3(a), (b) and (c) we show the eigenspectra associated with the 3-, 5- and 7-state chain Λ configurations respectively as a function of P with $P_i = P$ and $C_i = C$ for all i. Note that in each Figure there is a state at zero energy, which corresponds to the ground state. By maintaining our system in this state, we prevent decoherence.

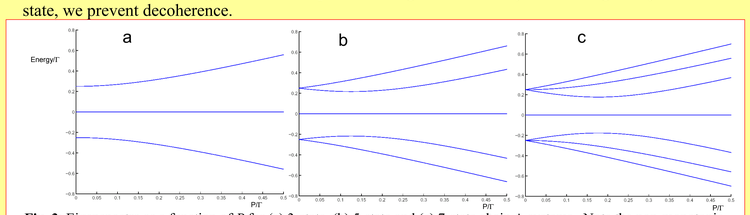

Fig. 3: Eigenspectra as a function of P for (a) 3-state, (b) 5-state and (c) 7-state chain Λ systems. Note the non-monotonic progression of the innermost dressed states on (b) and (c), and the presence of the ground state with Energy 0.

6. Probe Susceptibility

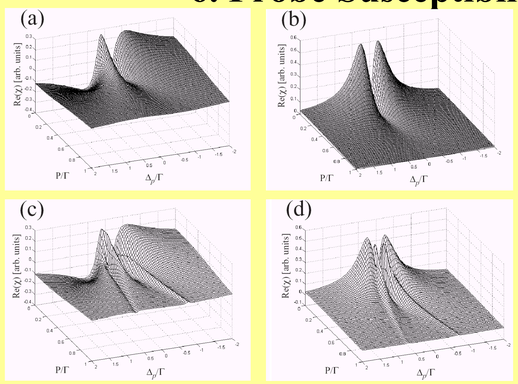

Fig. 6: Probe susceptibility (a) real and (b) imaginary for the 3-state chain Λ, and (c) real and (d) imaginary susceptibilities for the 5-state chain Λ.

We have also investigated probe susceptibility spectra (real and imaginary parts) using a density matrix approach for the 3-state and 5-state chain Λ systems. These are presented in Fig. 6 and should be compared with the eigenspectra presented in Fig. 3.

5. Intensity Dependent Effects

To analytically investigate intensity dependent effects, we investigate the composition of the ground state. Unnormalized coefficients expressing this are presented in the table for 3-, 5- and 7-state chain Λ systems. From these coefficients we may easily determine the intensity dependent dispersion R

$$R^{(3)} = \frac{C^2}{\Omega^4},$$

$$R^{(5)} = \frac{C^2(\Omega^4 + 2P^2\Omega^2 - 2P^4)}{(\Omega^4 - P^2\Omega^2 + P^4)^2},$$

$$R^{(7)} = \frac{C^2(\Omega^8 + 4P^4\Omega^4 - 8P^6\Omega^2 + 4P^8)}{\Omega^4(\Omega^4 - 2P^2\Omega^2 + 2P^4)^2},$$

$$R^{(n)} = \frac{1}{P}\frac{\partial}{\partial\Delta_p}\left(\sum_{i=1}^{n}\rho_{g_i e_i}\right).$$

For a given material and configuration, it is possible to calculate an intensity dependent group velocity. Using standard values for Rubidium we have calculated values for the group velocity as a function of P/Γ, although it is more important to concentrate on the overall shapes of the graphs, rather than the absolute values. These clearly show a strong nonlinear effect, and increasing nonlinearity with increasing order of chain Λ.

	$\lvert\mathcal{D}_0^{(3)}\rangle$	$\lvert\mathcal{D}_0^{(5)}\rangle$	$\lvert\mathcal{D}_0^{(7)}\rangle$
α_{0,g_1}	$\frac{C}{P}$	$\frac{C^2}{P^2}$	$-\frac{C^3}{P^3}$
α_{0,e_1}	$\Delta_p\frac{C}{\Omega^2}$	$\Delta_p\frac{C^2(\Omega^2+P^2)}{P(\Omega^4-C^2P^2)}$	$\Delta_p\frac{C(\Omega^2-P^2)(\Omega^2+2P^2)}{\Omega^2(\Omega^4-C^2P^2)}$
α_{0,g_2}	-1	$-\frac{C}{P}$	$\frac{C^2}{P^2}$
α_{0,e_2}		$\Delta_p\frac{C(2\Omega^2-P^2)}{\Omega^4-C^2P^2}$	$\Delta_p\frac{2C^2\Omega^2}{P(\Omega^4-C^2P^2)}$
α_{0,g_3}		1	$-\frac{C}{P}$
α_{0,e_3}			$\Delta_p\frac{C(\Omega^4+2C^4)}{\Omega^2(2C^2P^2-\Omega^4)}$
α_{0,g_4}			1

Unnormalized coefficients of the ground state, $|D_0\rangle$ for 3-, 5- and 7-state chain Λ systems.

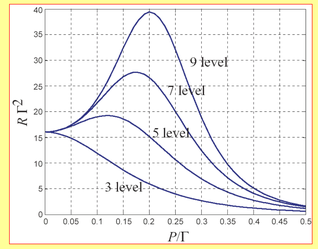

Fig. 4: Graphs showing dispersion (times linewidth squared) $R\Gamma^2$ as a function of P/Γ with $C/\Gamma = 0.25$ for 3-, 5-, 7-, and 9-state chain Λ systems.

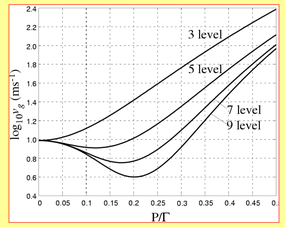

Fig. 5: Graphs showing $\log_{10}$ of group velocity, v_g, as a function of P/Γ with C/Γ = 0.25 for 3-, 5-, 7-, and 9-state chain Λ systems.

For a given material and configuration, it is possible to calculate an intensity dependent group velocity. Using standard values for Rubidium we have calculated values for the group velocity as a function of P/Γ, which are presented in Fig. 5, although it is more important to concentrate on the overall shapes of the graphs, rather than the absolute values. These clearly show a strong nonlinear effect, and increasing nonlinearity with increasing order of chain Λ.

Conclusions

We have shown that there exist interesting nonlinear properties for chain Λ atoms, and in particular, we have focussed on the intensity-dependent dispersion as a measure for these nonlinear optical properties. The nonlinearity of these systems increases as the number of atomic states increases, whilst the EIT transparency is maintained. It therefore appears likely that such multistate systems will be useful in the search for new quantum nonlinear optical materials. Our analysis has been confined to the optically thin regime. Clearly, in a full study, which would include intensity dependent group velocities in such highly nonlinear media, it is important to understand propagation effects and especially the effect of such high nonlinearities on pulse shape. Such analysis goes beyond our simple picture and will be the focus of the future work.

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