

# Conditions for spin squeezing in a cold $^{87}\text{Rb}$ ensemble

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## Abstract

We study the conditions for generating spin squeezing via a quantum non-demolition measurement in an ensemble of cold  $^{87}\text{Rb}$  atoms. By considering the interaction of atoms in the  $5S_{1/2}(F = 1)$  ground state with probe light tuned near the  $D_2$  transition, we show that, for large detunings, this system is equivalent to a spin-1/2 system when suitable Zeeman substates and quantum operators are used to define a pseudo-spin. The degree of squeezing is derived for the rubidium system in the presence of scattering causing decoherence and loss. We describe how the system can decohere and lose atoms, and predict as much as 75% spin squeezing for atomic densities typical of optical dipole traps.

**Keywords:** spin squeezing, quantum non-demolition measurement, rubidium, ensemble, scattering

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

There has recently been much interest in coupling light with atomic ensembles to develop a quantum interface. Several proposals have been published for utilizing this kind of coupling for spin squeezing [1–3], quantum memories [4], quantum teleportation [5], entanglement [6], magnetometry [7] and atomic clocks [8]. Many of these proposals have been realized experimentally using samples of alkali atoms in vapour cells and in magneto-optical traps (MOT) [9–14]. Spin squeezing is the simplest of these applications, and is often regarded as a benchmark of the light–atomic ensemble interaction. It has been demonstrated a few times: first in a MOT by mapping squeezed states of light onto the atomic spin [9], then in vapour cells via a quantum non-demolition (QND) measurement [10], and recently using the same method in a MOT but with the help of feedback [14].

In this article we study the conditions for generating spin squeezing via a QND measurement [2] in a cold ensemble of  $^{87}\text{Rb}$  atoms using the  $5S_{1/2}(F = 1)$  ground state of the  $D_2$  transition. We show that this system is formally equivalent to a spin-1/2 system when suitable Zeeman substates and quantum operators are used to define a pseudo-spin. In this scheme, we

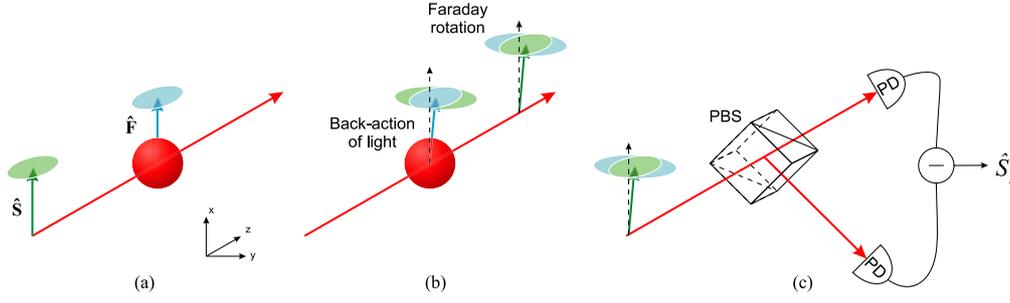
expect to have a higher light–atomic ensemble coupling than in previous work even when possible sources of decoherence and loss arising from these choices are identified and taken into account.

This article is organized into five sections. The next section describes the interaction between atoms and light considered here. Section 3 shows how the complicated  $^{87}\text{Rb}$  system can be reduced to an effective spin-1/2 system. In section 4, we calculate the degree of squeezing attainable in the presence of decoherence and loss. Finally, we present the conclusions in section 5.

## 2. Spin-squeezing interaction

Spin squeezing can be created by using a polarized off-resonant pulse of light to perform a QND measurement of the spin [2]. First, the Stokes vector  $\hat{\mathbf{S}}$  (polarization) of the probe pulse and the spin vector  $\hat{\mathbf{F}}$  of the atomic ensemble are prepared in a coherent state pointing in the  $x$  direction (figure 1(a)). As we send the pulse through the sample, the light and atoms interact via the dipole interaction, which in such schemes is described by the Hamiltonian

$$\hat{H}_{SS} = \hbar\Omega\hat{S}_z\hat{F}_z, \quad (1)$$



**Figure 1.** Spin-squeezing interaction: (a) preparation of the initial coherent states; (b) light–atom dipole interaction and entanglement of quantum fluctuations; and (c) polarimetric measurement of the probe light using a polarizing beam splitter (PBS) and a couple of photodetectors (PD).

where  $\Omega$  is a coupling strength,  $\hat{S}_z$  is the  $z$  component of the Stokes vector  $\hat{\mathbf{S}}$  of light and  $\hat{F}_z$  is the  $z$  component of the atomic spin vector  $\hat{\mathbf{F}}$ . In this interaction, the polarization of the light is rotated due to the Faraday effect and there is a back-action of the light onto atoms which rotates the orientation of the spin (figure 1(b)), and at the same time, their quantum fluctuations become entangled [15]. If this interaction acts for a time  $\tau$ , then for small  $\Omega\tau$ , it produces the following relation between the fluctuations of  $\hat{\mathbf{S}}$  and  $\hat{\mathbf{F}}$  [16]:

$$\begin{aligned}\delta\hat{F}_y^{\text{out}} &= \delta\hat{F}_y^{\text{in}} + \Omega\tau \langle \hat{F}_x \rangle \delta\hat{S}_z^{\text{in}}, \\ \delta\hat{F}_z^{\text{out}} &= \delta\hat{F}_z^{\text{in}}, \\ \delta\hat{S}_y^{\text{out}} &= \delta\hat{S}_y^{\text{in}} + \Omega\tau \langle \hat{S}_x \rangle \delta\hat{F}_z^{\text{in}}, \\ \delta\hat{S}_z^{\text{out}} &= \delta\hat{S}_z^{\text{in}}.\end{aligned}\quad (2)$$

As can be seen, a measurement of  $\hat{S}_y^{\text{out}}$  with a polarimeter (figure 1(c)) contains information about the spin component  $\hat{F}_z$ . This QND measurement leads to squeezing of the fluctuations  $\delta\hat{F}_z$ . If we ignore decoherence and loss mechanisms, the degree of squeezing has been shown [6, 17–19] to be

$$\xi^2 = \frac{1}{1 + \rho_0\eta}, \quad (3)$$

with  $\rho_0$  being the resonant optical density and  $\eta$  the integrated spontaneous emission rate (number of photons scattered per atom over a probe pulse). The degree of squeezing is defined such that  $\xi^2 = 1$  for a coherent state and  $\xi^2 < 1$  for a squeezed state.

### 3. Reduction of the <sup>87</sup>Rb system to an effective spin-1/2 system

The ideal case of a spin-1/2 system as the one depicted in figure 2(a) is simple to consider [2]. In this system, the  $\sigma^+$  and  $\sigma^-$  modes of the field interact with four-level atoms of spin 1/2. After adiabatically eliminating the excited states, this interaction is described by an interaction Hamiltonian of the form (1), resulting in the typical relations (2) between  $\hat{\mathbf{S}}$  and  $\hat{\mathbf{F}}$ . Finally, we can squeeze the atomic spin by performing a QND measurement through a measurement of  $\hat{S}_y^{\text{out}}$ .

When realizing this kind of interaction in a realistic system like Rb, one has to consider a more complicated, high spin

number system, and therefore reformulate the problem. In our particular case of <sup>87</sup>Rb, the lowest spin number is 1 (see figure 2(b)).

One possible realization in <sup>87</sup>Rb is to use a coherent superposition of the  $|5S_{1/2}, F = 1, m_F = -1\rangle$  and  $|5S_{1/2}, F = 1, m_F = +1\rangle$  levels ( $|-\rangle$  and  $|+\rangle$  from now on) as shown in figure 2(b). In this case, the chosen quantum observables are components of the alignment tensor, namely  $\hat{T}_x = \hat{F}_x^2 - \hat{F}_y^2$  and  $\hat{T}_y = \hat{F}_x\hat{F}_y + \hat{F}_y\hat{F}_x$ , and  $\hat{F}_z$  [20]. In fact

$$\begin{aligned}\hat{T}_x &= |-\rangle \langle +| + |+\rangle \langle -|, \\ \hat{T}_y &= i(|-\rangle \langle +| - |+\rangle \langle -|), \\ \hat{F}_z &= |+\rangle \langle +| - |-\rangle \langle -|.\end{aligned}\quad (4)$$

We can now define a collective pseudo-spin  $\hat{\mathbf{J}}$  by

$$\begin{aligned}\hat{J}_x &\equiv \frac{1}{2} \sum_k \hat{T}_x^k, \\ \hat{J}_y &\equiv \frac{1}{2} \sum_k \hat{T}_y^k, \\ \hat{J}_z &\equiv \frac{1}{2} \sum_k \hat{F}_z^k,\end{aligned}\quad (5)$$

where the superscript  $k$  denotes the single-atom operators and we sum over all atoms. This definition fulfils the angular momentum commutation relations

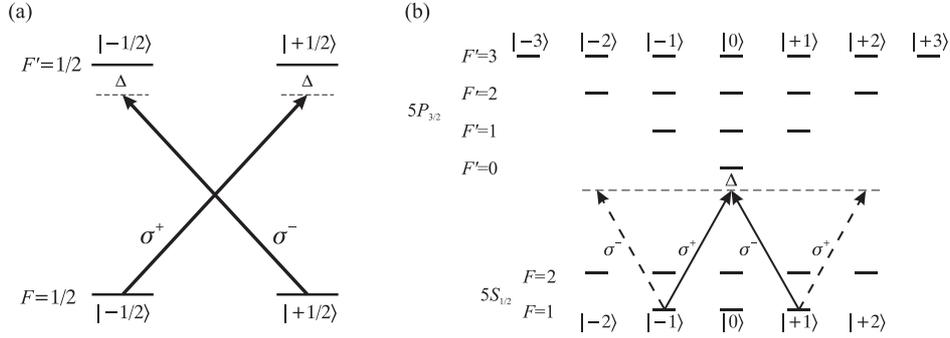
$$[\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk} \hat{J}_k, \quad (6)$$

when  $F = 1$ , where  $\epsilon_{ijk}$  is the Levi-Civita tensor. Hence, one could squeeze the pseudo-spin  $\hat{\mathbf{J}}$  along the  $z$  axis, as in the ideal case, if a QND-type interaction (1) exists between  $\hat{\mathbf{J}}$  and  $\hat{\mathbf{S}}$ .

This interaction can be derived as follows. Consider the dipole interaction Hamiltonian for an off-resonant field [17, 21]

$$\hat{H}_{\text{int}} = \sum_{F, F'} \hat{\mathbf{E}}^{(-)} \cdot \frac{\hat{\alpha}_{F, F'}}{\hbar \Delta_{F, F'}} \cdot \hat{\mathbf{E}}^{(+)}, \quad (7)$$

where  $\hat{\mathbf{E}}^{(\pm)}$  are the positive and negative frequency field operators of the probe field,  $\Delta_{F, F'}$  is the detuning of the probe from the  $F \rightarrow F'$  transition of the D<sub>2</sub> line in <sup>87</sup>Rb and  $\hat{\alpha}_{F, F'}$  is the atomic polarizability tensor of the transition. The latter is a rank-2 spherical tensor, which can be decomposed into the direct sum of a scalar, a vector and a tensor term:  $\hat{\alpha}_{F, F'} =$



**Figure 2.** QND interaction between the atomic spin and the light polarization in (a) an ideal spin-1/2 system and (b) the  $^{87}\text{Rb}$  system (solid arrows show the effective spin-1/2 system).

$\hat{\alpha}_{F,F'}^{(0)} \oplus \hat{\alpha}_{F,F'}^{(1)} \oplus \hat{\alpha}_{F,F'}^{(2)}$ . This leads to the decomposition of the interaction Hamiltonian into  $\hat{H}_{\text{int}} = \hat{H}^{(0)} + \hat{H}^{(1)} + \hat{H}^{(2)}$ , which can be expressed as [17]

$$\hat{H}^{(0)} = \alpha_0 g \sum_{F'} \frac{\alpha_{F,F'}^{(0)}}{\Delta_{F,F'}} \hat{n} \hat{N}, \quad (8a)$$

$$\hat{H}^{(1)} = \alpha_0 g \sum_{F'} \frac{\alpha_{F,F'}^{(1)}}{\Delta_{F,F'}} \hat{S}_z \hat{J}_z, \quad (8b)$$

$$\hat{H}^{(2)} = \alpha_0 g \sum_{F'} \frac{\alpha_{F,F'}^{(2)}}{\Delta_{F,F'}} \left[ \hat{S}_x \hat{J}_x - \hat{S}_y \hat{J}_y + \frac{2\hat{n}\hat{N}}{\sqrt{6}} \right], \quad (8c)$$

when we consider the transitions from the ground states  $|F = 1, m_F = \pm 1\rangle$  to all the excited states  $F'$ . Here  $\alpha_{F,F'}^{(i)}$  is the rank- $i$  unitless polarizability coefficient of the  $F \rightarrow F'$  transition,  $\hat{n}$  ( $\hat{N}$ ) is the photon (atom) number operator and

$$\alpha_0 = \frac{3\epsilon_0 \hbar \Gamma \lambda_0^3}{8\pi^2}, \quad (9)$$

$$g = \frac{\omega_0}{2\epsilon_0 V},$$

with  $\Gamma$  being the spontaneous decay rate,  $\lambda_0$  ( $\omega_0$ ) the transition wavelength (frequency) and  $V$  the interaction volume. The components of the Stokes vector can be expressed in terms of the annihilation and creation operators of the  $\sigma^\pm$  modes of the field:

$$\hat{S}_x = \frac{1}{2} (\hat{a}_-^\dagger \hat{a}_+ + \hat{a}_+^\dagger \hat{a}_-),$$

$$\hat{S}_y = \frac{i}{2} (\hat{a}_-^\dagger \hat{a}_+ - \hat{a}_+^\dagger \hat{a}_-), \quad (10)$$

$$\hat{S}_z = \frac{1}{2} (\hat{a}_+^\dagger \hat{a}_+ - \hat{a}_-^\dagger \hat{a}_-),$$

which can be related to the field operators  $\hat{\mathbf{E}}^{(\pm)}$ .

As can be seen from (8a),  $\hat{H}^{(0)}$  is proportional to  $\hat{n}\hat{N}$  and can be viewed as a global phase shift common to the two polarization modes of the probe pulse, which does not produce any signal on the output of the polarimeter, and therefore can be omitted. The term  $\hat{S}_x \hat{J}_x - \hat{S}_y \hat{J}_y$  in the rank-2 Hamiltonian (8c) corresponds to Raman transitions between  $|+\rangle$  and  $|-\rangle$ ; the other term is also proportional to  $\hat{n}\hat{N}$ , and can be omitted. Furthermore, for the case of detunings much larger than the hyperfine splitting of the excited states ( $|\Delta_{F,F'}| \gg |\Delta_{\text{hfs}}^e|$ ), the sum over  $\alpha_{F,F'}^{(2)}$  tends to zero, and hence  $\hat{H}^{(2)}$  can be

neglected. A similar situation occurs for  $\hat{H}^{(1)}$  in (8b), but this time only the contributions from  $F' = 1, 2$  tend to zero when  $|\Delta_{F,F'}| \gg |\Delta_{\text{hfs}}^e|$ , leaving just those from  $F' = 0$ .

Altogether, we are left with an effective three-level  $\Lambda$  system formed by the  $|\pm\rangle$  ground states and the  $F' = 0$  excited state (solid arrows in figure 2(b)), which does not exhibit Raman transitions, and therefore is formally equivalent to the simple spin-1/2 system of figure 2(a). The remaining effective Hamiltonian is

$$\hat{H}_{\text{eff}} = \alpha_0 g \frac{\alpha_{1,0}^{(1)}}{\Delta_{1,0}} \hat{S}_z \hat{J}_z, \quad (11)$$

which is of the form (1) necessary for the QND interaction.

#### 4. Degree of squeezing in the presence of scattering

We now consider the degree of squeezing for a general spin- $F$  system. The degree of squeezing defined by Wineland *et al* [22] for a frequency standard based on the Ramsey method is

$$\xi^2 \equiv \frac{\langle (\Delta \hat{F}_z)^2 \rangle}{|\langle \hat{F}_x \rangle|^2} 2NF, \quad (12)$$

which implies entanglement between the individual atomic spins when there is squeezing [23].

In the presence of scattering, atoms can be lost when they are pumped out of the initial atomic system, or can undergo decoherence when they stay within the system. These two cases are discussed in the following subsections, where we denote by  $\beta$  the number of scattered photons which produce loss, and by  $\gamma$  those that produce decoherence, with  $\eta = \beta + \gamma$ .

##### 4.1. Atom loss and decoherence

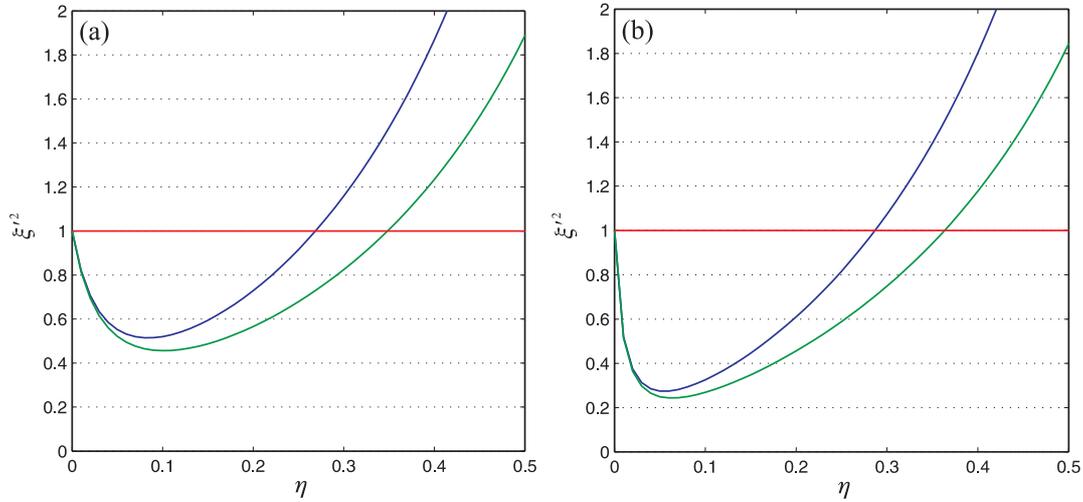
The case of atom loss occurs when the atoms are pumped out of the initial atomic system due to e.g. collisions with the background or spontaneous decay into a state not participating in the interaction.

In this case, it can be shown [24] that the variance  $\langle (\Delta \hat{F}_z')^2 \rangle$  of the remaining  $N' = (1 - \beta)N$  atoms is

$$\langle (\Delta \hat{F}_z')^2 \rangle = (1 - \beta)^2 \langle (\Delta \hat{F}_z)^2 \rangle + \beta(1 - \beta)N \frac{F}{2}. \quad (13)$$

Using equation (12), the degree of squeezing for the remaining atoms is

$$\xi'^2 = (1 - \beta)\xi^2 + \beta. \quad (14)$$



**Figure 3.** Degree of squeezing  $\xi^2$  as a function of the integrated scattering rate  $\eta$  for (a) a standard MOT ( $\rho_0 = 25$ ) and (b) a standard FORT ( $\rho_0 = 100$ ), and for a coherent state ((red) dotted curve), the spin-1/2 system ((blue) solid curve) and the  $^{87}\text{Rb}$  system ((green) dashed curve).

We now assume that  $\gamma N$  atoms undergo decoherence due to scattering within the initial coherent superposition. The total variance of  $\hat{F}'_z$  in this case is transformed in the same way as (13), but with the added variance  $\text{var}(\hat{F}'_z)_\gamma$  of the individual decohered atoms:

$$\langle(\Delta\hat{F}'_z)^2\rangle = (1-\gamma)^2\langle(\Delta\hat{F}_z)^2\rangle + \gamma(1-\gamma)N\frac{F}{2} + \gamma N\text{var}(\hat{F}'_z)_\gamma, \quad (15)$$

and the degree of squeezing will be

$$\xi^2 = \xi^2 + \frac{\gamma}{1-\gamma} + \frac{2\text{var}(\hat{F}'_z)_\gamma}{F} \frac{\gamma}{(1-\gamma)^2}. \quad (16)$$

#### 4.2. Spin-1/2 and $^{87}\text{Rb}$ systems

If we now consider the spin-1/2 system of figure 2(a), we notice that in this scheme, atoms can only undergo decoherence due to photon scattering ( $\gamma = \eta, \beta = 0$ ). Hence, the degree of squeezing can be promptly calculated from (3) and (16), and taking into account that the variance of each of the decohered atoms is  $\text{var}(\hat{F}'_z)_\gamma = 1/4$ . Figure 3 shows the calculated degree of squeezing as a function of the integrated scattering rate ((blue) solid curves) for (a) a standard MOT with  $\rho_0 = 25$  and (b) a typical far off-resonant trap (FORT) with  $\rho_0 = 100$ .

The case of  $^{87}\text{Rb}$  is again more complicated. Assuming that the probe light interacts only with the  $|F = 1\rangle$  ground state, i.e.  $|\alpha_{1,F'}^{(i)}/\Delta_{1,F'}| \gg |\alpha_{2,F'}^{(i)}/\Delta_{2,F'}|$ , if the atoms decay into the  $|F = 2\rangle$  hyperfine ground state, they do not interact with the light any more. Furthermore, if they decay in the  $|F = 1, m_F = 0\rangle$  Zeeman substate, they interact with the two polarization modes equally (see figure 2(b)) and do not produce any signal at the polarimeter. Hence all these atoms have fallen out of the pseudo-spin system considered, and from the perspective of the light-atom interaction, these atoms are simply lost. On the other hand, if the atoms are excited and decay back to the relevant states  $|\pm\rangle$  they will reduce the coherence of the system.

Generalizing the expressions above for the pseudo-spin  $\hat{\mathbf{J}}$  and putting (3), (14) and (16) together, we can arrive to

the following expression for the degree of squeezing in the presence of decoherence and loss:

$$\xi^2 = \frac{1-\beta}{1+\rho_0\eta} + \eta \frac{1-\beta}{1-\eta} + \gamma \frac{1-\beta}{(1-\eta)^2}, \quad (17)$$

where we have used  $\text{var}(\hat{J}'_z)_\gamma = 1/4$ . In our  $^{87}\text{Rb}$  system,  $\gamma = \frac{5}{3}\beta$ , according to the branching ratios that determine how  $\eta$  splits. This expression is shown as dashed (green) curves in figure 3. Notice that although this system is an effective spin-1/2 system, it outperforms the ideal spin-1/2 system, illustrating the fact that spin squeezing is more robust against loss than against decoherence, as can be seen by comparing equations (13) and (15).

As is shown in figure 3, for a given value of  $\rho_0$  there is an optimum value of  $\eta$  for which  $\xi^2$  is minimum. This arises from the fact that although scattering produces decoherence and loss, a certain degree of scattering is needed for the atoms to interact with the probe light. As much as 55% squeezing can be achieved for  $\eta = 0.10$  in a standard MOT ( $\rho_0 = 25$ ) and 75% for  $\eta = 0.06$  in a typical FORT ( $\rho_0 = 100$ ).

## 5. Conclusion

We have presented a scheme for  $^{87}\text{Rb}$  to perform spin squeezing of an atomic ensemble via a QND measurement and compared it to an ideal spin-1/2 system. We have found that the rubidium system can be reduced to an effective spin-1/2 system for large detunings ( $|\Delta_{F,F'}| \gg |\Delta_{\text{hfs}}^e|$ ) by considering the different tensor components of the atomic polarizability and choosing suitable optical polarization and atomic states, and with the help of a pseudo-spin defined in terms of the alignment tensor.

The degree of squeezing is derived for the rubidium system in the presence of scattering causing decoherence and loss, showing that it is more robust against loss than against decoherence. We describe how the system can decohere and lose atoms, and identify a minimum in  $\xi^2$  for a given value of  $\rho_0$ , which arises from the competition between the destructive scattering of photons and the desirable coupling between light and atoms. As much as 75% squeezing at  $\eta = 0.06$  is predicted for a typical FORT with  $\rho_0 = 100$ .

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