

Multiparticle State Tomography: Hidden Differences

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We address the problem of completely characterizing multiparticle states including loss of information to unobserved degrees of freedom. In systems where nonclassical interference plays a role, such as linear-optics quantum gates, such information can degrade interference in two ways, by decoherence and by distinguishing the particles. Distinguishing information, often the limiting factor for quantum optical devices, is not correctly described by previous state-reconstruction techniques, which account only for decoherence. We extend these techniques and find that a single modified density matrix can completely describe partially coherent, partially distinguishable states. We use this observation to experimentally characterize two-photon polarization states in single-mode optical fiber.

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The development of techniques for characterizing pure and mixed quantum states has enabled many advances in quantum information and related fields. Whether in order to study the effects of decoherence [1], to optimize the performance of quantum logic gates [2], to quantify the amount of information obtainable by various parties in quantum communications protocols [3], or to adapt quantum error correction protocols to real-world situations [4], it is first necessary to obtain as complete a characterization as possible of the state of a given quantum system (or ensemble). In the general case of mixed states, this involves reconstruction of a density matrix, a mathematically complete description of the degrees of freedom of interest in a quantum system. Entanglement with experimentally inaccessible or “hidden” degrees of freedom (sometimes called “the environment”) enters the density matrix as a reduction in the off-diagonal coherences. In some quantum systems—those composed of multiple particles that cannot be individually addressed—reduced coherence can only partially describe the effects of hidden degrees of freedom. Another phenomenon, distinguishability, arises when hidden degrees of freedom provide information that could in principle be used to tell the particles apart without necessarily leading to any changes in the coherences of the density matrix. This Letter will show how a density matrix characterization of such states can be performed while taking into account the decoherence and distinguishability as distinct phenomena.

Systems of particles that cannot be individually addressed occur commonly in quantum optics and elsewhere. A central example is the Hong-Ou-Mandel (HOM) effect [5], in which nonclassical interference causes photon bunching at a beam splitter. The effect results in photons with the same characteristics entering the same mode, making it impossible to individually address the photons, i.e., to manipulate or measure them individually. Many other major results in the field of quantum optics such as

the generation of Bell states, the demonstration of teleportation [6], linear-optics quantum computing [7], the generation of cluster states [8] and the demonstration of quantum logic gates [9,10] also use nonclassical interference and necessarily involve states with indistinguishable, nonindividually-addressable photons.

Numerous experiments have directly studied the properties of nonindividually-addressable photons [11–13]. The work of Bogdanov *et al.* showed that the polarization state of two photons forms a controllable three-level system or qutrit suitable for many protocols in quantum information [3] and quantum cryptography [14,15] and proposes a method for performing state tomography to characterize the density matrix of the qutrit state. This characterization is done under the *assumption* that the two photons in the state are indistinguishable, an assumption that is justified by measurement of high visibility HOM-like [5] two-photon interference. If that assumption were invalid the tomography method would give an incorrect description of the state.

All photons are, of course, indistinguishable in the fundamental sense of obeying Bose-Einstein statistics. We are concerned with another, operational sense of “distinguishable,” often encountered in discussions of multiphoton coherence. For example, the HOM effect acts on photons which arrive simultaneously at a beam splitter, but not on photons which arrive separated by more than their coherence time. We say that the photons could in principle be distinguished by their arrival times, and thus do not interfere. When we refer to distinguishability and indistinguishability in this Letter it is this kind of distinguishing information that we have in mind. Photons can be characterized by numerous degrees of freedom including arrival time, frequency, propagation direction, position, transverse mode and polarization. These degrees of freedom may or may not be experimentally accessible, depending on the capabilities of the experimental apparatus. In general, we

describe as “visible” those degrees of freedom that can be measured by a given apparatus, and as “hidden” those that cannot. This paper will explore the effect that distinguishability in hidden degrees of freedom due to timing information, for instance, can have on measurements performed on a visible degree of freedom such as polarization.

Our approach can be used on a variety of photon states such as those created by combining nonorthogonal photon modes [16], those that combine the output from different spontaneous parametric down-conversion sources [11] and those generated by stimulated parametric down-conversion [17]. Although this Letter treats systems of two photons, the same technique can be readily extended to a larger number of photons. This extension will be treated in another Letter [18].

If one starts from the assumption that the photons are indistinguishable in the hidden degrees of freedom as in [11] then one will use a 3×3 matrix to describe a polarization qutrit as opposed to the 4×4 matrix used for distinguishable photons [19]. For a more general description one could allow for a distinguishing degree of freedom and explore what limitations the fact that it is hidden places on measuring density matrix elements.

A general pure state of two photons can always be written as a superposition of tensor products of states in the visible and hidden degrees of freedom

$$\psi = \sum_i c_i |\phi_i\rangle_{\text{vis}} |\chi_i\rangle_{\text{hid}}, \quad (1)$$

where $|\phi_i\rangle_{\text{vis}}$, $|\chi_i\rangle_{\text{hid}}$ are eigenstates of exchange operators \mathbf{X}_{vis} and \mathbf{X}_{hid} for the visible and hidden degrees of freedom, respectively, with the same eigenvalue ± 1 . The requirement that the whole state be bosonic so that $\mathbf{X}_{\text{vis}} \otimes \mathbf{X}_{\text{hid}} |\psi\rangle = |\psi\rangle$ guarantees that each term can be written with the visible and hidden parts of the state either both symmetric or both antisymmetric. A completely general state of two photons is a mixture of states such as $|\psi\rangle$, described by a density matrix $\rho = \sum_j w_j |\psi_j\rangle \langle \psi_j|$.

A reduced density matrix describing only the visible degrees of freedom can be obtained by tracing out the hidden degrees of freedom in ρ . We define $\rho_{\text{vis}} = \text{Tr}_{\text{hid}}[\rho]$ which is sufficient to describe any measurement outcome on the visible degrees of freedom. Any such outcome has a corresponding operator $\mathbf{B} = \mathbf{B}_{\text{vis}} \otimes \mathbb{1}_{\text{hid}}$, where \mathbf{B}_{vis} acts only on the visible degrees of freedom and $\mathbb{1}_{\text{hid}}$ is the identity operator on the hidden degrees of freedom. Expectation values can be written as

$$\begin{aligned} \mathbf{B} &= \text{Tr}[\rho(\mathbf{B}_{\text{vis}} \otimes \mathbb{1}_{\text{hid}})] = \text{Tr}_{\text{vis}}[\text{Tr}_{\text{hid}}(\rho(\mathbf{B}_{\text{vis}} \otimes \mathbb{1}_{\text{hid}}))] \\ &= \text{Tr}_{\text{vis}}[\rho_{\text{vis}} \mathbf{B}_{\text{vis}}]. \end{aligned} \quad (2)$$

We define projectors $\mathbf{P}_{S,A}$ onto the symmetric (S) and antisymmetric (A) subspaces of the Hilbert space for the visible degrees of freedom. Using $\mathbf{P}_S + \mathbf{P}_A = \mathbb{1}$ and expanding ρ in terms of bosonic states as in Eq. (1) it can be shown that ρ_{vis} has the property that

$$\rho_{\text{vis}} = \mathbf{P}_S \rho_{\text{vis}} \mathbf{P}_S + \mathbf{P}_A \rho_{\text{vis}} \mathbf{P}_A. \quad (5)$$

This means that ρ_{vis} contains no coherences between the symmetric and antisymmetric subspaces, unlike distinguishable-particle density matrices [19]. We now specialize to the experimental situation in which the visible degree of freedom is polarization. The symmetric space is spanned by $\{|H_1 H_2\rangle, |\psi^+\rangle, |V_1 V_2\rangle\}$ and the antisymmetric space by $|\psi^-\rangle$, where $|\psi^\pm\rangle \equiv (|H_1 V_2\rangle \pm |V_1 H_2\rangle)/\sqrt{2}$. ρ_{vis} will be written in this basis.

Physically the lack of coherences between symmetric and antisymmetric states expresses lack of information about the labeling of the photons. This is illustrated by the distinguishable-particle states $|H_1 V_2\rangle$, $|V_1 H_2\rangle = (|\psi^+\rangle \pm |\psi^-\rangle)/\sqrt{2}$, for which the phase between $|\psi^+\rangle$ and $|\psi^-\rangle$ carries the information about the ordering of the photons. When this ordering is unmeasurable the magnitude of the coherence must be zero, as in ρ_{vis} . Nevertheless, the populations of $|\psi^-\rangle$ and $|\psi^+\rangle$ are measurable. For the same reason all of the coherences between the symmetric and antisymmetric states in ρ_{vis} are zero. Thus ρ_{vis} divides naturally into two submatrices, a 3×3 submatrix for the symmetric subspace and a 1×1 submatrix for the antisymmetric subspace as shown below:

$$\rho_{\text{vis}} = \begin{pmatrix} \begin{pmatrix} \rho_{\text{HH,HH}} & \rho_{\text{HH},\psi^+} & \rho_{\text{HH,VV}} \\ \rho_{\psi^+,\text{HH}} & \rho_{\psi^+,\psi^+} & \rho_{\psi^+,\text{VV}} \\ \rho_{\text{VV,HH}} & \rho_{\text{VV},\psi^+} & \rho_{\text{VV,VV}} \end{pmatrix} & 0 \\ 0 & (\rho_{\psi^-,\psi^-}) \end{pmatrix}. \quad (4)$$

If all the population is contained in the symmetric subspace then one recovers exactly the 3×3 matrix that would be measured under the assumption of indistinguishable photons. If, however, there is population in the antisymmetric state then the presence of distinguishing information in the hidden degrees of freedom can be inferred. Distinguishing information can only be detected when it correlates to polarization. Measuring all the population to be in $|H_1 H_2\rangle$, for instance, reveals no information about whether the two photons are distinguishable because ordering and polarization are uncorrelated [20]. In such circumstances the distinguishing information is undetectable by any apparatus that is only sensitive to polarization, but by the same token it is irrelevant to the outcome of any measurements made with that apparatus.

By tomographic measurement of the visible density matrix ρ_{vis} we characterize several different experimentally produced two-photon polarization states in polarization-maintaining single-mode fiber. As shown in Fig. 1, a 50-fs pulsed Ti:sapphire laser centered at 810 nm was frequency doubled to 405 nm, pumping a spontaneous parametric down-conversion crystal and creating pairs of photons. The crystal was phase matched in a type-II collapsed-cone geometry [21], so that the photons were not polarization entangled, but rather emerged in separate horizontally (H) and vertically (V) polarized beams. These beams were recombined into a single mode on a polarizing

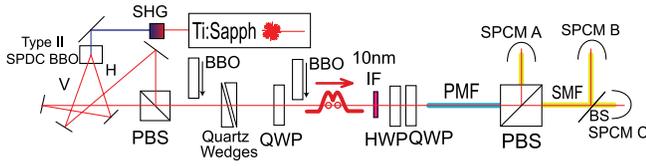


FIG. 1 (color online). Experimental implementation of state preparation and tomography protocols showing polarizing beamsplitters (PBS), beamsplitters (BS), nonlinear BBO crystals, a second harmonic generation crystal (SHG), quarter waveplates (QWP), half wave plates (HWP), polarization-maintaining fiber (PMF), single-mode fiber (SMF), and single photon counting modules (SPCM). A spontaneous parametric down-conversion (SPDC) crystal produces pairs of H and V photons. The separation between the H and V photons is controlled with movable quartz wedges and very long delays can be introduced by inserting a thick piece of BBO into the beam. Single-mode fiber and a 10 nm interference filter make the photons essentially indistinguishable in the spatiotemporal modes. Tomography is performed with a set of wave plates and a polarizing beam splitter. This system can implement all the measurements in Table I.

beam splitter, creating a two-photon state. A set of matched quartz wedges was used to fine-tune the delay between the H and V photons. Also, a 3-mm-thick piece of β -Barium Borate (BBO) could be inserted anywhere in the beam path to create a large birefringent delay. This birefringent delay introduced distinguishing timing information that was inaccessible to our detectors, which were not sensitive on the 100 fs time scale of the delay. This was our hidden degree of freedom. By controlling the delay between the pulses, the degree of overlap could be varied from completely overlapped to completely separated. Preparation wave plates inserted before and after the quartz wedges allowed various polarization states to be created. Interference filters and single-mode fiber served to limit distinguishing information in the spatiotemporal mode degrees of freedom.

The set of projectors measured in our experiment is given in Table I. The detection apparatus consisted of a

TABLE I. The measurement operators implemented in the tomography experiment. The detectors can detect either a coincidence between two photons in the H mode thereby implementing the projector $\mathbf{P}_{HH} \equiv |H_1 H_2\rangle\langle H_1 H_2|$ or a coincidence between the H and V modes thereby implementing $\mathbf{P}_{HV} \equiv |H_1 V_2\rangle \times \langle H_1 V_2| + |V_1 H_2\rangle\langle V_1 H_2|$. A quarter- and half-wave plate at angles q and h , respectively, placed before the detection apparatus, effectively rotate the detection operators to $U^{\otimes 2}\mathbf{P}(U^\dagger)^{\otimes 2}$, where \mathbf{P} is either \mathbf{P}_{HH} or \mathbf{P}_{HV} and $U \equiv \exp[i\pi(\sigma_z \cos 2h - \sigma_x \sin 2h)] \exp[\frac{i\pi}{2}(\sigma_z \cos 2q - \sigma_x \sin 2q)]$, where σ_x , σ_y and σ_z are the Pauli matrices.

h	q	\mathbf{P}	h	q	\mathbf{P}	h	q	\mathbf{P}
0°	0°	\mathbf{P}_A	22.5°	0°	\mathbf{P}_A	45°	0°	\mathbf{P}_A
22.5°	45°	\mathbf{P}_B	11.25°	0°	\mathbf{P}_A	0°	22.5°	\mathbf{P}_B
45°	22.5°	\mathbf{P}_A	22.5°	0°	\mathbf{P}_B	22.5°	22.5°	\mathbf{P}_A
0°	0°	\mathbf{P}_B						

polarizing beam splitter that projected the photons to either H or V and a set of single photon counting modules as shown in Fig. 1. A coincidence between detectors B and C measured \mathbf{P}_{HH} from Table I, while a coincidence between A and either B or C measured \mathbf{P}_{HV} . These two fundamental measurements were rotated with the measurement quarter-wave plate and half-wave plate to implement the full set of measurements listed in Table I.

The simplest state to prepare is composed of one horizontal and one vertical photon with a variable delay between them. When the H and V photons are overlapped as in Fig. 2(a), tomography shows that 98% of the population in the state is contained in the completely symmetric state $|\psi^+\rangle$ indicating that the two photons are highly indistinguishable. When the photons are delayed by a time larger than their coherence time we obtain Fig. 2(b). The population splits with 45% of the population in $|\psi^+\rangle$ and 55% of the population in $|\psi^-\rangle$, indicating that the photons are completely distinguishable to within the experimental limits of our measurement. When the delay is less than the coherence time we obtain the state in Fig. 2(c) with 62% of the population in $|\psi^+\rangle$ and 31% of the population in $|\psi^-\rangle$ indicating partial hidden distinguishability.

The most widely investigated state of two indistinguishable photons is the 2-NOON state [NOON refers to a path-entangled state, $1/\sqrt{2}(|N, 0\rangle + |0, N\rangle)$] [16,22] which in terms of polarization is $|2:0\rangle_{H,V} \equiv (|2_H, 0_V\rangle + |0_H, 2_V\rangle)/\sqrt{2}$. The state $|1_H, 1_V\rangle$ is a 2-NOON state in the circular basis (to within a global phase) because of the creation operator relation $a_H^\dagger a_V^\dagger = i(a_L^\dagger a_L^\dagger - a_R^\dagger a_R^\dagger)/2$. Experimentally, a quarter-wave plate can map the circular basis onto the H-V basis and turn $|1_H, 1_V\rangle$ into $(2_H, 0_V + 0_H, 2_V)/\sqrt{2}$.

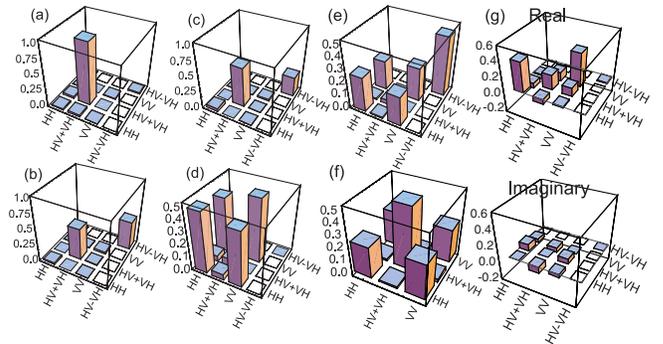


FIG. 2 (color online). The density matrices measured with quantum state tomography. The imaginary parts of (a) through (f), which are not shown, had all elements less than 0.05. White elements are inaccessible to measurement. (a) Indistinguishable H and V photons, (b) distinguishable H and V photons, (c) partially distinguishable H and V photons, (d) indistinguishable photons transformed to the 2-NOON state, (e) distinguishable photons with the same transformation applied, (f) the same state as it would be characterized by the technique of [12] (g) a 2-NOON state after passage through decohering, misaligned BBO crystal.

When the indistinguishable state in Fig. 2(a) is converted to a 2-NOON state in this way, all the population remains confined to the symmetric subspace and the generated state, Fig. 2(d), is indeed a reasonable approximation to the 2-NOON state with a fidelity of 0.90 and a concurrence of 0.54. On the other hand when the same transformation is applied to the state with hidden distinguishability in Fig. 2(b) the state in Fig. 2(e) is generated. The fidelity of this density matrix to the NOON state is only 0.49, nearly identical to the 0.5 fidelity that an incoherent mixture of HH and VV states would have to the NOON state. Its concurrence is zero. In a multiphoton interference experiment such as that in [16] Fig. 2(d) would display high visibility interference fringes whereas the state from Fig. 2(e) would not. One might expect a tomography protocol that assumes indistinguishable photons, such as that proposed in [12], to break down when confronted with a state such as Fig. 2(d). To check this we used the density matrix in Fig. 2(d) to calculate the outcomes of the measurements taken in [12] and linearly reconstructed an indistinguishable photon density matrix. The resulting matrix [Fig. 2(f)] mistakenly puts all the $|\psi^-\rangle$ population in the $|\psi^+\rangle$ state. This density matrix will incorrectly predict the outcome of measurements made in other bases such as the diagonal basis.

The state in Fig. 2(g) is obtained if a 2-NOON state is made from indistinguishable photons and then sent through a complicated nonunitary process implemented by inserting a thick piece of BBO whose axis is at a small angle relative to the horizontal. As can be seen, the BBO reduces the size of the coherence between the HH and VV terms (as well as causing some rotation), but leaves the $|\psi^-\rangle$ term unaffected. This demonstrates that decoherence can occur without introducing distinguishability between the photons. Comparison with Fig. 2(e) shows that decoherence and distinguishability are distinct effects with different experimental signatures.

We have developed and demonstrated experimentally state tomography of two-photon polarization states, including, for the first time, the effects of distinguishing information in hidden degrees of freedom. Such distinguishing information destroys nonclassical interference, is often a limiting factor in linear-optics devices such as quantum gates, and is not correctly described by previous tomography schemes, which account only for decoherence. Our tomography technique produces a “visible density matrix” which predicts the outcome of all polarization measurements, and which describes the effects of both decoherence and distinguishability. This was demonstrated clearly with our production and measurement of a “NOON” state affected by both decoherence and distinguishability. The approach can be applied to other types of states and extended to larger numbers of particles.

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