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Orbital angular momentum correlations of entangled paired photons

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Abstract

The generation of paired photons entangled in the spatial degree of freedom, i.e., in orbital angular momentum, offers a convenient physical resource to investigate the nature of entanglement in a multidimensional Hilbert space with controllable dimensionality. The two main physical processes that generate pairs of photons which show correlations in orbital angular momentum are (a) spontaneous parametric down-conversion (SPDC), and (b) Raman transitions induced in atomic ensembles. One question naturally arises: what kinds of correlations exist between the orbital angular momentum of the generated photons? The answer might be different if we consider the whole quantum state of the generated photons, i.e., all possible directions where the pairs of photons can be emitted, or if we consider only a small section of the full set of directions.

Keywords: orbital angular momentum, parametric down conversion, Raman transitions, quantum optics and entanglement

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The orbital angular momentum (OAM) of entangled photons is increasingly being used as a resource for the implementation of quantum information algorithms that, either inherently live in a Hilbert space of dimensions higher than two (qudits), or exhibit enhanced efficiency in increasingly higher dimensions (see [1] and references therein). These include the demonstration of the violation of bipartite, three-dimensional Bell inequalities [2], the implementation of the so called quantum coin tossing protocol with qutrits [3], and the generation of quantum states in ultra-high-dimensional spaces [4].

1.1. Spontaneous parametric down-conversion

All of the experiments mentioned above use as the source for generating paired photons with entangled properties, spontaneous parametric down-conversion (SPDC). In this process, an intense beam pumps a nonlinear crystal, where with a low probability, a pair of lower frequency photons is generated (see figure 1). Photons are known to be emitted in cones whose shape depends on the phase matching conditions inside the nonlinear crystal. Most of the relevant experiments reported up to now make use of a small section of the full down-conversion cone. Different sections of the down-conversion cone can thus be explored by relocating the single-photon counting modules.

Several experiments [5–8] seem to support the validity of the selection rule \( \ell_p = \ell_s + \ell_i \), where \( \ell_p \) is the OAM per photon of the classical pump beam, and \( \ell_s \) and \( \ell_i \) are the winding numbers of the modes into which the quantum state of the signal and idler photons are projected, respectively. In other words, only signal and idler photons that fulfils the above-mentioned selection rule can be detected. Some other experiments, while not directly measuring the OAM of the down-converted photons, demonstrate the existence of ellipticity of the spatial waveform [9–11], which should make possible the detection of photons with \( \ell_p \neq \ell_s + \ell_i \). Under some restrictive conditions, the selection rule \( \ell_p = \ell_s + \ell_i \) can be derived from first principles [12–14], although, as will
nearly collinear (emitted \[18, 22, 23\]. In most cases, such detection modes are
set of directions where the Stokes/anti-Stokes photons can be
anti-Stokes photons are detected in a small section of the full
(OAM) \[21\] degrees of freedom.

In this scheme, as shown in figure 2, a classical pump beam
generates entangled pairs of photons, in the last few years,
Although SPDC is by far the most widely used source for
1.2. Raman transitions in atomic ensembles

Although SPDC is by far the most widely used source for
generating entangled paired photons, in the last few years,
another interesting scheme has been proposed that makes use
of atomic ensembles to generate entangled pairs of photons.
In this scheme, as shown in figure 2, a classical pump beam
(the WRITE beam) impinges on an ensemble of \( N \) atoms,
for instance, rubidium or cesium, and it induces the emission
of, at most, a single photon (Stokes photon) from one of the
atoms \[18\]. Such emission generates a collective atomic excitation that can be read by a control beam, which induces
the emission of another photon (anti-Stokes photon) correlated
with the Stokes photon. Quantum correlations mediated
by the generation of a collective excitation in an ensemble
of atoms have been observed in polarization \[19\], in the
time–frequency \[20\], and in the orbital angular momentum
(OAM) \[21\] degrees of freedom.

In a typical experimental configuration, the Stokes and
anti-Stokes photons are detected in a small section of the full
set of directions where the Stokes/anti-Stokes photons can be
emitted \[18, 22, 23\]. In most cases, such detection modes are
nearly collinear (\(~2^\circ–3^\circ\) with the direction of propagation of
the counter-propagating pump and control beams \[24\]. But
other situations can be considered as well, as in the case of
transverse emitting configurations, where the Stokes/anti-Stokes photons propagate transversally to the pump/control
beams \[25\].

Experiments reported up to now \[21, 26\] show that the
selection rule \( m_p = m_c = m_s = m_{as} \) is fulfilled, where \( m_p \)
and \( m_c \) are the OAM per photon of the classical pump and
control beams, and \( m_s \) and \( m_{as} \) are the winding numbers of the
modes into which the quantum state of the Stokes and anti-
Stokes photons are projected. Notwithstanding, when more
general non-collinear configurations are considered \[27\], this
selection rule seems to be violated.

Since many quantum information schemes are based on
the existence of specific quantum correlations between pairs
of photons, the characterization of such correlations is very
important. The OAM correlations should be addressed in two
complementary scenarios, so that in each scenario the sough-
tafter OAM correlations can be different. In one scenario, the
spatial properties of all of the pairs of photons generated are
considered \[12\]. In this case, the OAM correlations can be
modified by the presence of any effect that breaks the azimuthal
symmetry around the pump beams that mediate the generation
of the paired photons. In another scenario, which is relevant
for current experimental applications, a small section of the full
downconversion cone is considered. Under this condition, the
use of a non-collinear configuration, or the presence of spatial
walk-off or any other symmetry-breaking perturbation, can
greatly modify the OAM correlations observed. In particular,
paired photons generated in different directions of emission
show correspondingly different spatial quantum correlations
and amount of entanglement.

This paper is divided into three sections. In section 2 we
discuss how to describe the OAM of single-, and two-photon,
quantum states. In section 3, we analyze the OAM correlations
of the two-photon state, when all possible directions of emission of the generated photons are considered. Finally, in section 4, the OAM correlations of paired photons generated in collinear, or nearly collinear, configurations are discussed.

2. The orbital angular momentum of single and paired photons

2.1. Single photons

The spatial properties of a single-photon quantum state $|\Psi\rangle$ are described by a mode function $\Phi$, so that

$$|\Psi\rangle = \int dp \Phi(p)a^\dagger(p)|0\rangle$$

where $|0\rangle$ is the vacuum state, $p = (p_x, p_y)$ is the transverse wavevector, and $a^\dagger(p)$ is the creation operator of one photon with transverse wavevector $p$. We assume that the generated photon is narrowband, with frequency $\omega$, and $p$ is restricted to a finite number of modes, or it can consist of an infinite, but discrete, number of modes.

Any mode function with an arbitrary amplitude profile can be expanded into spiral harmonic modes, so that it can be written as

$$\Phi(\rho, \varphi) = \sum_{m=-\infty}^{\infty} a_m(\rho) \exp(im\varphi)$$

where $\rho = (p_x^2 + p_y^2)^{1/2}$ and $\varphi = \tan^{-1} p_y/p_x$ are cylindrical coordinates in transverse wavevector space.

Mode functions which are not represented by a pure spiral harmonic mode correspond to photons in a superposition state, with the weights of the quantum superposition dictated by the harmonic mode that describes mathematically the mode function is $|C_m|^2$. The OAM content of the quantum state is then given by the array $P_m = |C_m|^2$. The value of $C_m$ is given by $C_m = \int dp \rho |a_m(\rho)|^2$, where

$$a_m(\rho) = \frac{1}{\sqrt{2\pi}} \int d\varphi \Phi(\rho, \varphi) \exp(-im\varphi).$$

Photons that are described by a superposition of OAM states can be prepared in a variety of ways. Such superpositions can be restricted to a finite number of modes, or it can consist of an infinite, but discrete, number of modes.

Within the paraxial regime of light propagation, any classical beam with an arbitrary amplitude profile can be expanded into Laguerre–Gauss (LG) modes, so that the amplitude $A$ of the electric field at $z = 0$ can be written as

$$A(p_x, p_y, z = 0) = \sum_{m=-\infty}^{\infty} \sum_{p=0}^{\infty} C_{mp} U_{mp}(p_x, p_y)$$

where $C_{mp} = \int dp A(p)U^*_{mp}(p)$. The functions $U_{mp}$ are Laguerre–Gauss (LG) modes. The index $p$ is the number of non-axial radial nodes of the mode and the index $m$, referred to as the winding number, describes the helical structure of the wavefront around a phase dislocation. When the amplitude is a pure LG mode with index $m$, the mode is an eigenstate of the OAM operator with eigenvalue $mh$.

When we consider photons that propagate in different directions, as will be the case here, there might appear some confusion about how to designate the OAM state of photons. In order to be more specific, let us consider a photon that propagates in the +z direction, therefore the direction of the linear momentum vector is also +z. If the mode function of the photon is a LG beam with index $m > 0$, the OAM vector $\vec{L}_z$ has the same direction as the momentum vector, as shown in figure 3(a). If the mode function of the photon is a LG beam with index $m < 0$, the OAM vector has the opposite direction to the linear momentum vector (see figure 3(b)). If the photon reverses its direction of propagation, and therefore its linear momentum, for instance by being reflected in a perfect mirror, the OAM vector does not reverse its direction [29], and the index $m$ that describes mathematically the mode function is still the same as before the photon was reflected, but it is clear that the reflected photon is different to the incident photon.

What is relevant physically is not the sign of $m$, which depends on the direction of propagation of the photon, but the relationship between the linear momentum and the OAM, so we can distinguish two types of photons, in relation to its OAM: photons where linear momentum and OAM vectors point out in the same direction, as shown in figures 3(a) and (d); and photons where the linear momentum and OAM vectors show opposite directions (figures 3(b) and (c)). Both types of photons can be easily distinguished by making them interfere with a plane wave that propagates in the same direction, resulting in a pattern of interference with a fork-like structure, inverted or not, depending on the type of photons present.

To avoid any confusion, we will always consider appropriate coordinate systems for each photon, where the +z axis is always given by the linear momentum of the photon, therefore by its direction of propagation. In this way,
the electric field (\(E_{in}\)) of the incident photon on the mirror, which propagates along the \(z_1 = z\) direction, can be written as

\[
E_{in}(x_1, z_1, t) = \int dp\ U_{m p}(p) \exp(ik z_1 + ip \cdot x_1 - i \omega t).
\]

(5)

while the electric field of the reflected photon (\(E_{out}\)), which propagates along the direction \(z_2 = -z\), can be written as

\[
E_{out}(x_2, z_2, t) = \int dp\ U_{-m p}(p) \exp(ik z_2 + ip \cdot x_2 - i \omega t).
\]

(6)

When we write both electric fields in the common coordinate system \((x, z)\), and we take into account the relationships between the coordinate systems \((x_1, z_1)\) and \((x_2, z_2)\), i.e., \(x_1 = x_2 = x\) and \(y_1 = -y_2 = y\), we obtain

\[
E_{in}(x, z, t) = \int dp\ U_{m p}(p) \exp(ik z + ip x + ip y - i \omega t)
\]

(7)

\[
E_{out}(x, z, t) = \int dp\ U_{-m p}(p) \exp(-ik z + ip x - ip y - i \omega t)
\]

showing explicitly the change of the OAM state (from \(+m\) to \(-m\)).

2.2. Pairs of photons

The quantum state of a two-photon pair is given by

\[
|\Psi\rangle = \int dp\ dq\ \Phi(p, q)a_{i}^{\dagger}(p)a_{i}^{\dagger}(q)|0\rangle_{s}|0\rangle_{i}
\]

(8)

where \(p\) and \(q\) are the transverse components of the signal and idler wavevectors, and \(a_{i}^{\dagger}\) are the corresponding creation operators for the signal and idler photons, respectively. One can decompose the mode function in the base of the eigenstates of the OAM operator as [14]

\[
\Phi(p, q) = \sum_{m_1 m_2} C_{m_1 m_2}(\rho_1, \rho_2) \exp(-im_1 \varphi_1 - im_2 \varphi_2).
\]

(9)

If the idler photon is projected into the quantum state \(|m_2, p_2\rangle\), whose mode function is a LG beam, the signal photon turns out to be

\[
|\Psi_s\rangle = \int dp\ \Phi_s(p)a_{i}^{\dagger}(p)|0\rangle_{s}
\]

(10)

with

\[
\Psi_s(p) = \int dq\ \Phi(p, q)U_{m_2 p_2}(q).
\]

(11)

The OAM content of the signal photon is given by the corresponding normalized array \(P_{m_1} = |C_{m_1}|^2\).

3. What kinds of correlations exist between the OAM of the generated photons?

3.1. SPDC: the full down-conversion cone

Let us consider a nonlinear crystal of length \(L\) (from \(z = -L/2\) to \(z = L/2\)), illuminated by a monochromatic laser pump beam propagating in the \(z\) direction, with frequency \(\omega_0\). The spatial shape of the pump beam at the center of the nonlinear crystal \((z = 0)\), in the transverse wavevector domain, can be written as

\[
E_{p}(p) = E_0(p_x + ip_y)\exp(-|p|^2 w_0^2/4),
\]

which corresponds to a beam which carries an OAM of \(m_p h\) per photon. \(E_0\) is a normalization constant, \(p = (p_x, p_y)\) is the transverse wavevector and \(w_0\) is the beam width. The signal and idler photons are assumed to be monochromatic, with \(\omega_s = \omega_i = \omega_p/2\), where \(\omega_s, i\) are the frequencies of the signal and idler photons. This is justified by the use of narrowband interference filters in front of the detectors.

We neglect the effect of the Poynting vector walk-off of the interacting beams. The angle of the down-conversion cone is assumed to be small, so that the polarization [30] and refractive index do not show noticeable changes with the direction of propagation. The nonlinear coefficient is assumed to be constant as well.

If we assume a coherent state for the pump beam, with coherent-state amplitude \(\xi_n\), the effective Hamiltonian in the interaction picture, can be written as

\[
H_{I}^{SPDC} = \epsilon_0 \int dV\ \chi_s^*(\epsilon_s - \epsilon_i)\epsilon_p + \text{h.c.}
\]

(12)

At first-order perturbation theory, the mode function \(\Phi\) of the two-photon quantum state is given by

\[
\Phi(P, Q) = E_0(P + Q)\text{sinc}(\Delta_s L/2)
\]

(13)

where \(P\) and \(Q\) are the transverse wavevectors for the signal and the idler. \(\Delta_s\) is given by \(\Delta_s = k_s(P + Q) - k_s(P) - k_s(Q)\), where the wavevectors write \(k_s(P) = [(\omega_s/c)^2 - |P|^2]^{1/2}\) with \((j = s, i, p)\), and \(n_j\) are the corresponding refractive index.

We can write \(|P + Q|^2 = \rho_s^2 + \rho_i^2 + 2\rho_s\rho_i\cos(\varphi_s - \varphi_i)\), where \(\rho_s = |P|,\) and \(\varphi_s = \tan^{-1} P_y/P_x\) are the modulus and phase of the transverse wavevector \(P\) in cylindrical coordinates. For the idler photon we have, similarly, \(\rho_i = |Q|\) and \(\varphi_i = \tan^{-1} Q_y/Q_x\). Therefore, one can write \(\text{sinc}(\Delta_s L/2) = \sum_{l=-\infty}^{\infty} H_i(\rho_s, \rho_i)\exp[i(\varphi_s - \varphi_i)]\). The pump beam can also be written as

\[
E_p(P + Q) = E_0 \exp\left\{-\left[\rho_s^2 + \rho_i^2 + 2\rho_s\rho_i\cos(\varphi_s - \varphi_i)\right]w_0^2/4\right\}
\]

\times \sum_{l=-m_p}^{m_p} \left|\frac{m_p}{l}\right|\rho_s^{|m_p-l|}\exp[i(l\varphi_s + i(m_p - l)\varphi_i)].
\]

(14)

The mode function given by equation (13) can thus be written as

\[
\Phi(P, Q) = \sum_{m=-\infty}^{\infty} G_m(\rho_s, \rho_i)\exp\left[im\varphi_s + i(m_p - m)\varphi_i\right]
\]

(15)

where the function \(G\) is determined by equation (14).

The main conclusion to be drawn from equation (15) is that, if polarization, refractive index and nonlinear coefficient show negligible azimuthal variations along the down-conversion cone, the OAM correlations of the spatial waveform of the biphoton state fulfill \(m_p = m_s + m_i\) [12]. Importantl,y this result requires considering the whole spatial waveform of the down-converted photons, i.e. the full down-conversion cone. Notwithstanding, these are not the OAM correlations that typical quantum information experiments based on spatial entanglement measure.
3.2. Raman transitions in atomic ensembles: all directions

We assume coherent monochromatic modes for the control and pump beams, with coherent-state amplitudes $\xi_\varepsilon$ and $\xi_p$, respectively. For non-resonant pump and control beams, the effective nonlinearity $\chi^{(3)}$ does not depend on the intensity of those beams [31]. The distribution of atoms in the cloud is assumed to be Gaussian, so the effective nonlinearity $\chi^{(3)}$ can be written as

$$\chi^{(3)}(x, y, z) \propto \exp \left[ -\frac{x^2 + y^2}{R^2} - \frac{z^2}{L^2} \right]$$ (16)

where $R$ is the size of the cloud of atoms in the transverse plane $(x, y)$ and $L$ is the size in the longitudinal direction.

The generated Stokes and Anti-Stokes photons are narrowband ($\sim$GHz) [20]; thus, the Stokes and anti-Stokes photons are assumed to be monochromatic, with $\omega_p + \omega_s = \omega_c + \omega_{as}$, where $\omega_{as}$ are the frequencies of the Stokes and anti-Stokes photons, and $\omega_{p,c}$ correspond to the frequencies of the pump and control beams.

The effective Hamiltonian in the interaction picture, that describes the photon–atom interaction, can be written as

$$H_1 = \epsilon_0 \int \! dV \, \chi^{(3)}(x, y, z) \, \varepsilon_\varepsilon \, \varepsilon_p \, \varepsilon_{as} + \text{h.c.}$$ (17)

The mode function $\Phi$ that describes the quantum state of the generated pair of photons, at first order of perturbation theory, is written as

$$\Phi(q_p, q_{as}) = \int \! \! dQ_p \, dQ_{as} \, E_p(Q_p) \, E_{as}(Q_{as}) \times \exp \left( -\Delta_1 R^2 / 4 - \Delta_2 s^2 / 4 - \Delta_3 c^2 L^2 / 4 \right)$$ (18)

where

$$\Delta_1 = Q_p^2 - Q_{as}^2 - Q_s^2 + Q_i^2$$
$$\Delta_2 = Q_{as}^2 - Q_s^2 + Q_{p,i}^2$$
$$\Delta_3 = k_p - k_c - k_s + k_{as}$$

and the longitudinal wavevector of the any of the interacting beams can be written as $k_i = [(\omega_i n_i/c)^2 - |Q_i|^2]^{1/2}$. $n_i$ is the refractive index at the corresponding wavelength, $|Q_i|^2 = (Q_i^p)^2 + (Q_i^s)^2$ and $c$ is the velocity of light in vacuum.

Let us define radial $\rho_i$ and azimuthal $\varphi_i$ coordinates $Q_i^p = \rho_i \cos \varphi_i$ and $Q_i^s = \rho_i \sin \varphi_i$ for $i = p, c, s, a, s$. The pump and control beams are written as Laguerre–Gauss beams, $E_p(\rho_p, \varphi_p) \propto \rho_p^{m_p} \exp(-\rho_p^2 w_p^2 / 4) \exp(i m_p \varphi_p)$ and $E_{as}(\rho_{as}, \varphi_{as}) \propto \rho_{as}^{m_{as}} \exp(-\rho_{as}^2 w_{as}^2 / 4) \exp(i m_{as} \varphi_{as})$, where $m_{p,c}$ and $m_{as}$ are the OAM quantum numbers of the beams, and $w_{p,c}$ are the corresponding beam widths. $\Delta_0 = k_p - k_c - k_s + k_{as}$ depend only on the radial coordinates, and

$$\Delta_1^2 + \Delta_2^2 = \rho_p^2 + \rho_c^2 + \rho_s^2 + \rho_{as}^2 + 2 \rho_p \rho_c \cos \varphi_p + \varphi_c$$
$$- 2 \rho_p \rho_{as} \cos \varphi_p + \varphi_{as}$$
$$- 2 \rho_c \rho_{as} \cos \varphi_c + \varphi_{as}$$
$$+ 2 \rho_{as} \cos \varphi_c + \varphi_{as}$$

Equation (18) can thus be written as

$$\Phi(\rho_c, \varphi_c, \rho_{as}, \varphi_{as}) = \sum_{m=-\infty}^{\infty} \mathcal{F}_m(\rho_p, \rho_{as}) \times \exp \left[ im \varphi_c + i(m - m_p + m_c) \varphi_{as} \right]$$ (20)

where the function $\mathcal{F}_m$ comes from integrating over $\rho_p$ and $\rho_c$.

Notice that equation (20) is formally identical to equation (15). Both in the case of SPDC, and in the case of Raman transitions induced in atomic ensembles, the relationships $m_p = m_s + m_i$ and $m_p - m_c = m_s - m_{as}$ describe perfect correlations between the OAM state of each photon of the generated pair. The pump beam in one case, and the pump and control beams in the other case, establish a special direction, the $z$ axis. The existence of azimuthal symmetry around this axis is therefore reflected in the existence of perfect OAM correlations between the two photons. But anything that would break such symmetry, would manifest in the violation of such a selection rule. In SPDC configurations, this might be the presence of Poynting vector walk-off of some of the interacting waves [15, 16]. In paired photons generated in atomic ensembles, this might be a noncylindrical distribution of atoms in both transverse dimensions.

4. Orbital angular momentum correlations in collinear configurations

We refer as collinear configurations to those configurations where the generated photons co-propagate or counter-propagate with the pump and control beams that mediate the generation of the photons. In this case, the mode function that describes the spatial shape of the pairs of photons is formally identical to equation (13) for SPDC, or to equation (18) for Raman transitions. Therefore, the selection rules $m_p = m_s + m_i$ (for SPDC) and $m_p - m_c = m_s - m_{as}$ for Raman transitions in atomic ensembles apply as well [27, 32]. But notice that now we are considering paired photons generated in specific directions.

The equivalence does not necessarily apply to the weight of each mode in the corresponding OAM decomposition. Although the selection rules that determine the OAM correlations are the same, the probability of detecting particular OAM modes can change. In [14], it was shown how to engineer such weights for the case of SPDC, by controlling the pump beam width and the length of the nonlinear crystal.

As an example, let us consider a type II noncollinear collinear configuration. Such a configuration can be achieved, for instance, using a periodically poled KTP crystal, where all waves propagate along the $X$ axis of the nonlinear crystal, and the generated photons bear orthogonal polarizations. In such a collinear configuration ($\varphi = 0$), the spatial mode function of the biphoton can be written as [33]

$$\Phi(p, q) = E_p(p + q) \text{sinc} \left[ \frac{|p - q|^2 L}{4k_p^0} \right]$$ (21)
where $k_0^0 = k_p(p = 0)$. In this case, the relationship $m_p = n_s + m_i$ is fulfilled.

The phase matching function, $\text{sinc}(\Delta k L/2)$, can be approximated by an exponential function that has the same width at the $1/e^2$ of the intensity: $\text{sinc}(\Delta k x^2) \simeq \exp[-\Gamma (\Delta k x)^2]$, with $\Gamma = 0.455$. For the case of a Gaussian pump beam, equation (21) can thus be written as

$$\Phi(p, q) = \left[ \frac{w_0^2 \Gamma L}{\pi^2 k_p^0} \right]^{1/2} \exp \left[ -\frac{p + q}{w_0^2} \right] \times \exp \left( -\frac{\Gamma L}{4k_p^0} |p - q|^2 \right).$$

(22)

Under these conditions, the OAM decomposition [14] of the mode function given by equation (22) is also the Schmidt decomposition of the mode function [34, 35], and is written as

$$\Phi(p, q) = \sum_{m, p} C_{mp} U_{mp}(p) U_{-mp}(q)$$

(23)

where

$$C_{mp} = (1 - z) e^{i m|p|/2}$$

(24)

with

$$z = \left[ \frac{w_0^2 - \Gamma L/k_p^0}{w_0^2 + \Gamma L/k_p^0} \right]^{1/2}$$

(25)

and $\sum_{m=\infty}^{\infty} \sum_{p=0}^{\infty} |C_{mp}|^2 = 1$. Notice that, under the Gaussian approximation, Schmidt modes are anti-correlated in the index $m$, but show perfect correlation in the index $p$. For $w_0 = \sqrt{\Gamma L/k_p^0}$ the pair of photons is not spatially entangled, i.e., only one term of the Schmidt decomposition ($m = 0$ and $p = 0$) remains. This is the appropriate working configuration when the goal is to generate pairs of photons non-entangled, and with a Gaussian shape. For typical values in PPKTP ($\lambda_p = 405 \text{ nm}$, a crystal length of $L = 30 \text{ mm}$ and refractive index $n_p \sim 1.7$), one obtains $w_0 \sim 22 \text{ \mu m}$.

5. Conclusion

When considering all of the possible directions of emission of the generated pairs of photons, the main conclusion to be drawn is that, if there is no source of azimuthal distinguishability with respect to the spatial direction set-up by the pump and control beams’ directions of propagation, the selection rules $m_p = m_a + m_i$ for SPDC, and $m_p - m_c = m_a - m_a$, for Raman transitions, are fulfilled. This is also the case for configurations where, although specific directions of emission are considered, the generated photons co-propagate or counter-propagate with the pump and control beams.

On the other hand, we should notice that the question of the total angular momentum conservation balance in SPDC requires the simultaneous consideration of the angular momentum of the electronic spins and orbitals, the crystalline structure of the nonlinear crystal and of the electromagnetic field [36, 37]. The analysis presented here might be an important step towards clarifying how angular momentum is effectively conserved, since to evaluate conservation laws, one should take into account all probability amplitudes that contribute to the quantum process.

In general, the azimuthal distinguishing information introduced in non-collinear configurations can affect the quantum properties of the photons generated, and cannot be neglected, even when other degrees of freedom of the photons are considered. This is the case, for instance, when the configurations considered here are used as sources of polarization-entangled photons. The azimuthal distinguishing information introduced by the direction of emission can affect the quantum properties of polarization-entangled photons when these photons are generated with different angles of emission [16]. This is the case when using two type I SPDC crystals whose optical axes are rotated $90^\circ$. This configuration, originally demonstrated for the generation of polarization-entangled photons [38, 39], has been used as well for the generation of hyperentangled quantum states [4].

It is also the case of the source considered in [40]. If the volume of interaction is spherical-like $(R \simeq L)$, the realization of high-dimensional entanglement by selecting several spatial modes (directions of emission), as proposed in [40], can be achieved without introducing spatial distinguishing information between different pairs of photons [16], which can degrade the quality of the entanglement generated. On the other hand, the presence of ellipticity of the mode function as a function of the emission angle, could restrict the angles of emission accessible for generating a polarization-entangled state with a degree of concurrence above a certain prescribed level.

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